Sets
http://gawron.sdsu.edu/semantics

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Overview

1. Sets
2. Subsets and powersets
3. Operations on sets

Outline

Sets

A **set** is a collection of things.

$$
\begin{aligned}
A &= \{\text{a, b, c}\}\\
B &= \{1, 2, 3\}\\
C &= \{\text{The Amazon River,}\\
&\quad\ \ \text{Donald Trump's left eyebrow,}\\
&\quad\ \ 3\}
\end{aligned}
$$

Membership

$$
\begin{aligned}
A &= \{\text{a, b, c}\}\\
B &= \{1, 2, 3\}\\
C &= \{\text{The Amazon River,}\\
&\quad\ \ \text{Donald Trump's left eyebrow,}\\
&\quad\ \ 3\}
\end{aligned}
$$

set membership: $\in$

$a \in A$

The opposite: $\notin$

Donald Trump $\notin$ C

Cardinality

$$\begin{aligned} A &= \{a, b, c\} \\ B &= \{1, 2, 3\} \\ C &= \{\text{The Amazon River}, \\ &\quad\ \text{Donald Trump's left eyebrow}, \\ &\quad\ 3\} \end{aligned}$$

The size of a set, called its **cardinality**, is written $|A|$:
$|A| = |B| = |C| = 3$

Which is true?
$|A| \in A$
$|A| \in B$
$|B| \in B$

Sets with cardinality 0

We allow sets with nothing in them, that is, sets with cardinality 0. We call such sets **empty sets** and write them as $\emptyset$. For any x, it is true that

$$x \notin \emptyset$$

Principle of extensionality

Two sets A and B are equal if they have the same members. That is if

$$A \subseteq B \text{ and } B \subseteq A$$

An immediate consequence is that we need to stop talking about **an** emptyset. There is only one: **the** empty set.

$$(\emptyset_1 \subseteq \emptyset_2 \;\; \& \;\; \emptyset_2 \subseteq \emptyset_1) \Leftrightarrow \emptyset_1 = \emptyset_2$$

The domain of discourse

The domain of discourse is the collection of things we're talking about.

$$\aleph = \{1, 2, 3, \ldots\}$$

$\aleph$ is called the set of **Natural Numbers.**

In arithmetic we call $\aleph$ the **universe of discourse**. This is sometimes called the set of **atoms**. The term atom contrasts with the term set. In set theory we talk mostly about sets, but the things we talk about that *aren't* sets are atoms. Atoms don't have members.

Semantics: domains of discourse

In semantics the domains of discourse is the things that language can be about, and language can be about **everything**. We will come up with a very special domain of discourse when we talk about possible worlds. Meanwhile, here are some sets that might naturally come up in semantics. See if you can guess what meanings they might be relevant for:

$$
\begin{array}{lcl}
[\![\text{dog}]\!] & = & \text{the set of all dogs} \\
[\![\text{toothbrush}]\!] & = & \text{the set of all toothbrushes} \\
[\![\text{hope}]\!] & = & \text{the set of all hopes} \\
[\![\text{walk}]\!]^{t} & = & \text{the set of all individuals that are} \\
 & & \text{walking at time t} \\
[\![\text{unicorn}]\!] & = & \text{the set of all unicorns} \\
[\![\text{goblin}]\!] & = & \text{the set of all goblins} \\
[\![\text{even odd}]\!] & = & \text{the set of all odd numbers exactly divisible by 2}
\end{array}
$$

Semantics: Principle of extensionality

$$
\begin{array}{lcl}
[\![\text{dog}]\!] & = & \text{the set of all dogs} \\
[\![\text{unicorn}]\!] & = & \text{the set of all unicorns} \\
[\![\text{goblins}]\!] & = & \text{the set of all goblins} \\
[\![\text{even odds}]\!] & = & \text{the set of all odd numbers exactly divisible by 2}
\end{array}
$$

Outline

Subset

$D \subset B$

$$B = \{1, 2, 3\}$$
$$D = \{1, 2, \}$$

Every D is a B!

If we want to include sets that are equal to B, we write $\subseteq$. So $D \subseteq B$ and $B \subseteq B$, but $B \not\subset B$ (compare $<$, $\leq$, and $\not<$ for numbers).

Note that the emptyset $\emptyset$ is a subset of every set, so in particular

$$\emptyset \subset B$$

Power sets

When we collect all the sets that are subsets of or equal to ($\subseteq$) some given set S we get a new set, a set of sets (more properly a **collection** of sets), called the power set of S, written $\mathcal{P}(S)$:

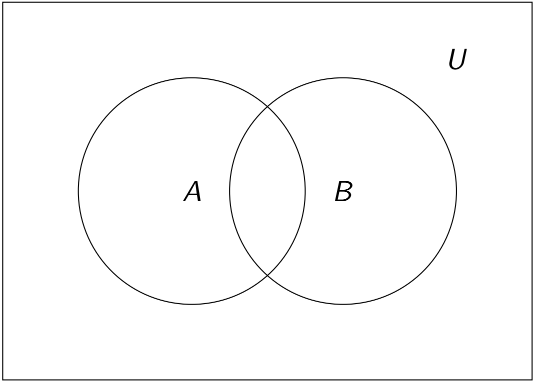
$$S = \{\text{a, b, c}\}$$
$$\mathcal{P}(S) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$$

$\emptyset$	0 element sets
$\{a\}, \{b\}, \{c\}$	1 element sets
$\{a, b\}, \{b, c\}, \{a, c\}$	2 element sets
$\{a, b, c\}$	3 element sets

Outline

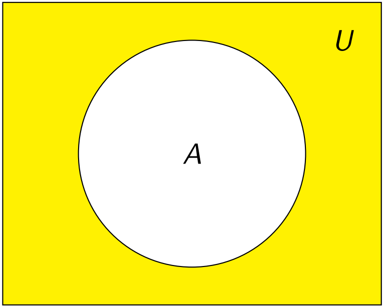
Two overlapping sets

Universe of discourse is U. Neither set is a subset of the other.



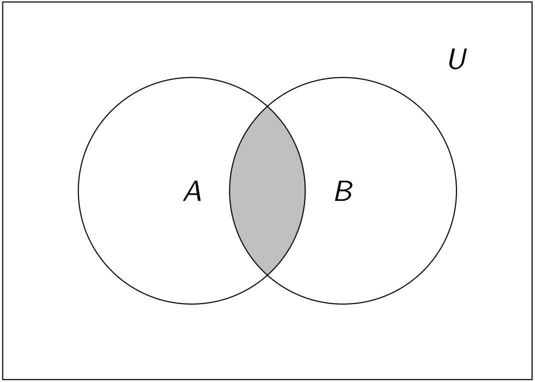
Complement of A

$$\overline{A}$$



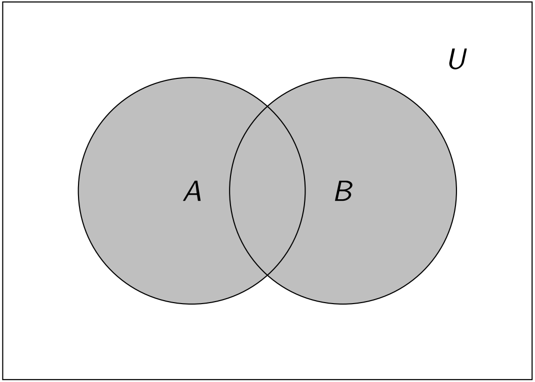
Set Intersection

$A \cap B$ the set of things in both A and B



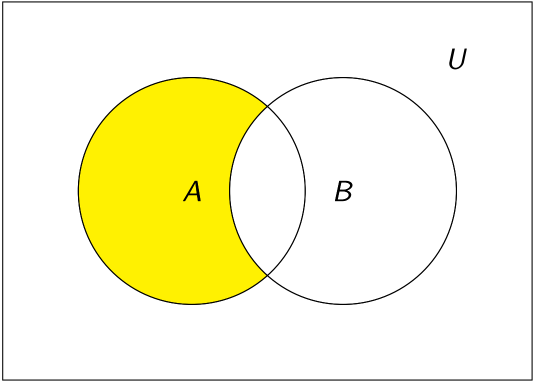
Set Union

$A \cup B$ the set of things in either A or B



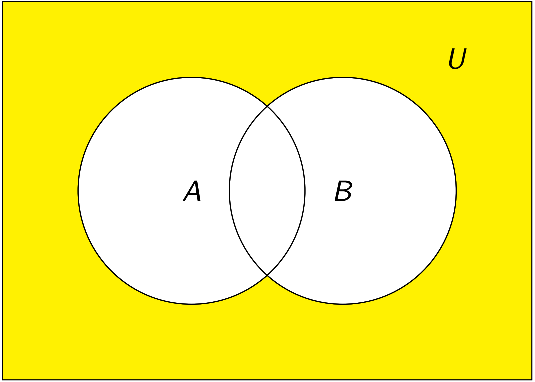
Set Difference

$A \setminus B$ the set of things in A that are **not** in B



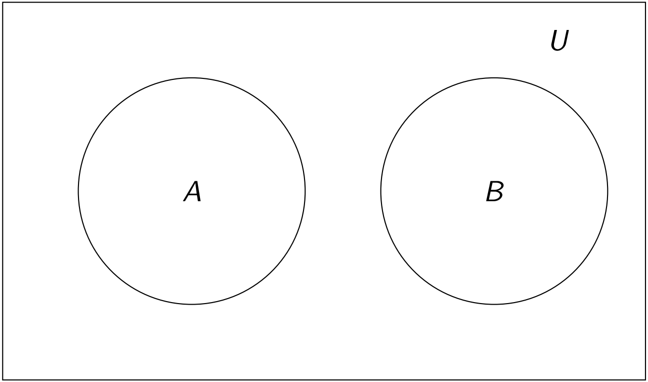
Complement of Union

$\overline{A \cup B}$ the set of things that are neither in A nor in B



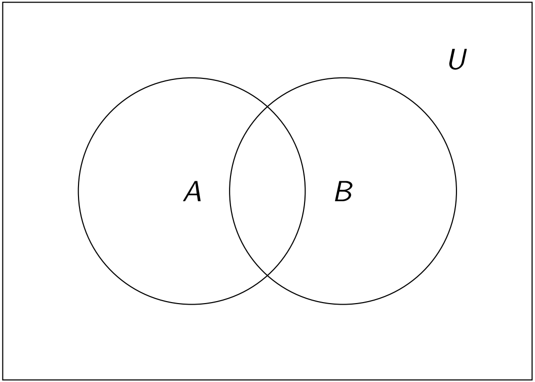
Two non-overlapping sets

$$
\begin{aligned}
A \cap B &= \emptyset \\
A \setminus B &= A
\end{aligned}
$$



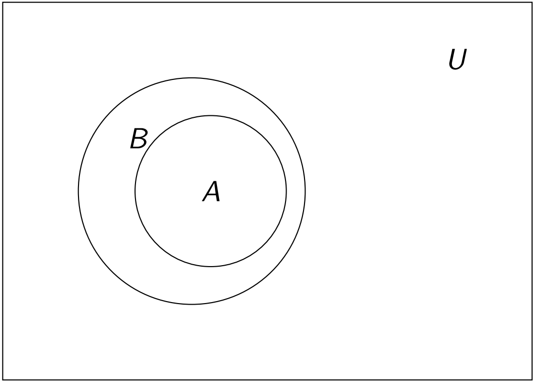
Two overlapping sets

$$
\begin{aligned}
A \cap B &\neq \emptyset \\
A \setminus B &\subset A
\end{aligned}
$$



Subsets

$$A \subset B$$
$$A \cap B \quad = \quad ?$$



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