# ***Logical Form & Predicate Logic***

Jean Mark Gawron
Linguistics
San Diego State University
gawron@mail.sdsu.edu
http://www.rohan.sdsu.edu/∼gawron

# Predicates and Arguments

# Logical Form

**The Logical Form** of English sentences can be represented by formulae of Predicate Logic,

What do we mean by the logical form of a sentence?

We mean: we can capture the **truth conditions** of complex English expressions in predicate logic, given some account of the denotations (extensions) of the simple expressions (words).

# The Leading Idea: Extensions

Nouns, verbs, and adjectives have predicates as their translations.

What does ’predicate’ mean?

’Predicate’ means what it means in predicate logic.

# ***Intransitive verbs***

Translating into predicate logic

(1) a. John walks.
    b. $\mathbf{walk}(\mathbf{j})$

(2) a. $[\![\mathbf{j}]\!]$ = John
    b. "The extension of 'j' is the individual John."
    c. $[\![\mathbf{walk}]\!] = \{x \mid x \text{ walks }\}$
    d. "The extension of 'walk' is the set of walkers"

(3) a. $[\![\textbf{John walks}]\!] = [\![\textbf{walk(j)}]\!]$
    b. $[\![\mathbf{walk}(\mathbf{j})]\!]$ = true iff $[\![\mathbf{j}]\!] \in [\![\mathbf{walk}]\!]$

# Extensional denotations

- Remember two kinds of denotation. For work with logic, denotations are always **extensions**.
- We call a denotation such as $[\![\mathbf{walk}]\!]$ (a set) an **extension**, because it is defined by the set of things the word *walk* describes or "extends over"
- The denotation $[\![\mathbf{j}]\!]$ is also extensional because it is defined by the individual the word *John* describes.
- Next we extend extensional denotation to transitive verbs, nouns, and sentences.

# Transitive verbs

(4) a. John loves Mary
b. $\textbf{love}(\mathrm{j},\ \mathrm{m})$

(5) a. $[\![\textbf{j}]\!]$ = John
b. $[\![\textbf{m}]\!]$ = Mary
c. $[\![\textbf{love}]\!] = \{\langle x,\ y\rangle \mid x$ loves $y\}$
d. “The extension of ’love’ is the set of pairs of individuals $x$ and $y$ such that $x$ loves $y$.”

(6) a. $[\![\textbf{John loves Mary}]\!] =$ True iff $\langle [\![\textbf{John}]\!],\ [\![\textbf{Mary}]\!]\,\rangle \in [\![\textbf{love}]\!]$
b. $[\![\textbf{love}(\textbf{j},\ \textbf{m})]\!] =$ true iff $\langle [\![\textbf{j}]\!],\ [\![\textbf{m}]\!]\rangle \in [\![\textbf{walk}]\!]$

# Nouns

- A simple noun predication

(7) a. Fido is a dog.

b. $\mathbf{dog}(\mathrm{f})$

- But what about ...?

(8) A dog barked.

Different meaning of *dog*? Hopefully not!

# Adjectives

- A simple adjectival predication

(9) a. Fido is happy.

b. $\mathbf{happy}(\mathrm{f})$

- But what about ...?

(10) A dog is happy.

Different meaning of *happy*? Hopefully not!

# Assuming both nouns and verbs are predicates

- Fido is a dog.

$$\text{dog}(\text{f})$$

- Fido barked.

$$\text{bark}(\text{f})$$

- A dog barked.

$$\text{dog}(x) \wedge \text{bark}(x)$$

BOTH predicates present: some non-specific $x$ is a dog and barked.

# Assuming both nouns and adjectives are predicates

- Fido is a dog.

$$\text{dog}(\text{f})$$

- Fido is happy.

$$\text{happy}(\text{f})$$

- A dog is happy.

$$\text{dog}(x) \wedge \text{happy}(x)$$

  BOTH predicates present: some non-specific $x$ is a dog and is happy.

# A new use of $\wedge$

(11) a. A dog barked.

b. dog$(x) \wedge$ bark$(x)$

c. a dog is happy.

d. dog$(x) \wedge$ happy$(x)$

- We're using $\wedge$ even though the word *and* hasn't occurred in either sentence.
- $\wedge$ is going to turn out to have a lot more uses in our logical translations than just as a translation of *and*
- Other sentential logical connectives will also turn up in surprising places

# Sentential Connectives (revisited)

We still use sentential connectives from statement logic for English sentententential connectives (where they work!)

(12) a. John doesn't love Mary
b. $\neg\mathbf{love}(\mathsf{j},\ \mathsf{m})$

(13) a. John loves Mary and Fred loves Sue.
b. $\mathbf{love}(\mathsf{j},\ \mathsf{m}) \wedge \mathbf{love}(\mathsf{f},\ \mathsf{s})$

# Relational Nouns

- Alex is Bill's henchman.

  What kind of a predicate does the noun *henchman* correspond to?

  a. $henchman(\text{a}) \wedge ??$

  b. $henchman(\text{a}, \text{ b })$

- Relational nouns: *friend, enemy, mother, father, brother, sister, husband, wife, owner, bottom, promise, blame*

# Logical Form and surface syntax

| | |
|---|---|
| Brigitte is taller than Danny. | ...is taller than... . |
| Alex is Bill's henchman. | ... is ...'s henchman. |
| Fiji is near New Zealand. | ... is near ... . |

$\textsc{taller}\ (\mathsf{b},\ \mathsf{d})$

$\textsc{henchman}\ (\mathsf{a},\ \mathsf{b})$

$\textsc{near}(\mathsf{f},\ \mathsf{n})$

# *Quantifiers*

# Preview

Our proposal thus far is on the right . . .

but

. . . but there are some serious problems treating negation.

. . . So we introduce **quantifiers**.

# Extending the proposal in a natural way

(14) a. John loves someone.
b. $\mathbf{Love}(\mathrm{j},\ x)$

# Other one-place predicates

(15) a. John drives a Buick.

b. $\mathbf{Drive}(\text{j},\ x) \wedge \mathbf{Buick}(x)$

# *The problem: negation*

(16) a. John doesn't drive a Buick.

b. $\neg[\textbf{Drive}(\mathrm{j},\ x) \wedge \textbf{Buick}(x)]$

Does this mean the right thing?

| |
|---|
| For some unspecific $x$, it's not the case both that $x$ is a Buick and John drives $x$. |

# Scope

- The meaning we've got for (16) is that there's some specific Buick (say, Fred's) that John doesn't drive.
- Maybe that's a reading for (16), but it's surely not the most natural one.
- The meaning we want: It is NOT the case that there's a Buick that John drives.
- The problem is that at the moment we havent even got a way of writing down the most natural reading, on which the **scope of the negation claim includes the existence claim**.

# Restating the problem with truth conditions

(16b) gives the wrong truth-conditions for (16a)

(17) a. Suppose **B11** and **B12** are both Buicks. John drives **B11** and John doesn't drive **B12**.

b. Then there is an $x$ such that it's not the case both that $x$ is a Buick and John drives $x$. Namely **B12**. While **B12** is a Buick John doesnt drive it.

c. So the logical formula (16b) comes out true in these circumstances.

d. But the English sentence (16a) is not true in these circumstances. John shouldn't be driving ANY Buicks, yet he's driving **B11**.

e. The logical formula (16b) misdescribes the truth conditions of (16a) .

f. This is the **semantic analogue** of the grammar mis-describing the grammaticality of a sentence.

# A Solution

(18) a. John drives a Buick.

b. $\exists x[\mathbf{Drive}(\mathsf{j},\ x) \wedge \mathbf{Buick}(x)]$

c. $\exists x[\mathbf{Drive}(\mathsf{j},\ x) \wedge \mathbf{Buick}(x)]$ is true iff there is some entity $\mathbf{b}$ such that $[\mathbf{Drive}(\mathsf{j},\ \mathbf{b}) \wedge \mathbf{Buick}(\mathbf{b})]$ is true.

d. True whenever John drives **any** entity that is a Buick

e. False only if there is NO entity that is a Buick that John drives

f. $[\![\exists x\phi(x)]\!] =$ true iff there is some entity $\mathbf{b}$ such that $[\![\phi(x)]\!]^{\mathbf{b}/x} =$ true

# Negation

(19) a. John doesn't drive a Buick.

b. $\neg\exists x[\mathbf{Drive}(\text{j},\ x) \wedge \mathbf{Buick}(x)]$

c. $\neg\exists x[\mathbf{Drive}(\text{j},\ x) \wedge \mathbf{Buick}(x)]$ is true iff it is not the case that there exists some entity $\mathbf{b}$ such that $[\mathbf{Drive}(\text{j},\ \mathbf{b}) \wedge \mathbf{Buick}(\mathbf{b})]$ is true.

d. Previously: $\neg[\mathbf{Drive}(\text{j},\ x) \wedge \mathbf{Buick}(x)]$ is true iff there exists some entity $\mathbf{b}$ such that it is not the case that $[\mathbf{Drive}(\text{j},\ \mathbf{b}) \wedge \mathbf{Buick}(\mathbf{b})]$ is true.

# Revising previous analyses

**Other Fixes**

(20) a. A dog is happy.

b. $\exists x[\mathbf{Dog}(x) \land \mathbf{Happy}(x)]$

c. A dog barked.

d. $\exists x[\mathbf{Dog}(x) \land \mathbf{Bark}(x)]$

e. Fido is a dog.

f. $\exists x[\mathbf{Dog}(x) \land x = \mathsf{f}]$

# *Scope: the new analysis*

- By introducing $\exists x$ we formally marked the **scope** of an existence claim.
- In $\exists x \phi(x)$, we call $\phi(x)$ the **scope of the existential**.
- In $\neg\phi$ we call $\phi$ the **scope of the negation.**
- $\neg\exists x[\text{drive}(\text{j}, x) \wedge \text{Buick}(x)]$

| Operator | Scope |
|---|---|
| $\exists x$ | $[\text{drive}(\text{j}, x) \wedge \text{Buick}(x)]$ |
| $\neg$ | $\exists x[\text{drive}(\text{j}, x) \wedge \text{Buick}(x)]$ |

- We say: **the scope of the negation is wider than the scope of the existential.**

# Every and All

We use $\forall x$ to mean “for all $x$”

(21) a. $[\![\forall x \phi(x)]\!]$ is true iff for every $x$, $\phi(x)$ is true.
b. So we need to look at a large number of cases; Each needs to turn out true.
c. How many cases? All of them. Every entity in the universe.
d. *Every dog is a mammal* is a claim about every dog, not every entity in the universe.
e. How do we represent that fact?

# Wrong semantics

(22) a. Every dog is a mammal.

b. $\forall x[\mathbf{Dog}(x) \wedge \mathbf{Mammal}(s)]$

c. This requires every entity in the universe to be a dog and a mammal.

d. Paraphrase: Everything is a dog and a mammal.

e. We make no distinction between the truth conditions of *every dog is a mammal* (true) and *Every mammal is a dog*. (false)

# Right semantics

(23) a. Every dog is a mammal.

$$\forall x[\mathbf{dog}(x) \rightarrow \mathbf{mammal}(s)]$$

For every $x$: if $x$ a dog, then $x$ is a mammal.

b. Every mammal is a dog.

$$\forall x[\mathbf{mammal}(x) \rightarrow \mathbf{dog}(s)]$$

For every $x$: if $x$ a mammal, then $x$ is a dog.

# *Working through the truth-conditions*

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | T | T |

- $x$ not a dog dog$(x)$ $\rightarrow$ mammal$(x)$ T

  F F

  F T

  $x$ a mammal and dog dog$(x)$ $\rightarrow$ mammal$(x)$ T

  T T

  $x$ a nonmammal and dog dog$(x)$ $\rightarrow$ mammal$(x)$ F

  T F

- For every individual, being a dog implies being a mammal.