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## 1 Introduction

We introduce the connectives of first-order logic.
We introduce predicates. And a very simple semantics for them.

## 2 Truth-Functional Connectives

### 2.1 And

|  | $p$ | $q$ | $p \& q$ |
| :---: | :---: | :---: | :---: |
| $(\mathrm{a})$ | T | T | T |
| (b) | T | F | F |
| (c) | F | T | F |
| (d) | F | F | F |

2.1.1 Abraham Lincoln was elected in 1860 and he was re-elected in 1864.
2.1.2 John picked up the apple and he ate it.
2.1.3 ? John ate the apple and he picked it up. [temporal order]
2.1.4 You take one more step and I'll shoot. [= If you take one more step, I'll shoot]

### 2.2 Or

|  | $p$ | $q$ | $p \vee q$ |
| :---: | :---: | :---: | :---: |
| $(\mathrm{a})$ | T | T | T |
| (b) | T | F | T |
| (c) | F | T | T |
| (d) | F | F | F |

2.2.1 He rented either a mid-size or an economy car. [If in fact he rented both, this is still true]
2.2.2 Either there's no bathroom in this house or it's on the second floor. [In fact both statement can't be simultaneously true, but that's not due to the meaning of or]
2.2.3 You can have either the white one or the red one. [intended meaning: but not both]

### 2.3 Material implication

Material implication is the name we'll use for $\rightarrow$.

|  | $p$ | $q$ | $p \rightarrow q$ |
| :---: | :---: | :---: | :---: |
| $(\mathrm{a})$ | T | T | T |
| (b) | T | F | F |
| (c) | F | T | T |
| (d) | F | F | T |

2.3.1 If John ate the apple, he'll be sick.
2.3.2 Antecedent: John ate the apple

### 2.3.3 Consequent: He'll be sick.

Claim made: In those circumstances where the first sentence is true, the second sentence is true. So the first two lines of the truth table make perfect sense. The claim is safe when both sentences are true, and it is clearly false when the

But what about when the first sentence is false. Well if he didnt eat the apple the claim is safe whether he's sick or not. This claim only guarantees that IF he ate the apple sickness follows. So if he didnt the claim is still "true" according to our truth conditions.

Question: How well does this accord with our intutuions about conditionals? Answer: Not very.

|  | Antecedent | Consequent | Conditional | Truth <br> Value |
| :--- | :--- | :--- | :--- | :--- |
| (a) | T | T | If 1960 was divisible by 5, then 1960 <br> was a leap year. | T |
| (b) | F | T | If Al Gore won the election of 2000, <br> George Bush won the election of <br> 2004. | T |
| (c) | T | F | If George Bush won the election of <br> 2004, Al Gore won the election of <br> 2000. | F |

(a) just seems false. (b) is weird; it's not clear what kind of communicative act is being performed. (c) can be true as an instance of the "If X, I'll eat my hat" construction.

Finally, consider:
(d) If you kick me again, I'll punch you.

Here the right truth table seems to be:

| $(d)$ | Antecedent | Consequent | Truth |
| :--- | :--- | :--- | :--- |
| Value |  |  |  |$|$|  | T | T |
| :--- | :--- | :--- |
| T | F |  |
|  | F | F |
|  | F | F |

Crucially, if you don't kick me and I punch you, I have at least seriously misled you, if not downright lied.

## 3 Statement Logic Classification of sentences

### 3.1 Tautologies

Sentences like $p \rightarrow(q \rightarrow p)$, parenthesized THIS way, are called tautologies, because they cant help but be true:

|  | $p$ | $q$ | $q \rightarrow p$ | $p \rightarrow(q \rightarrow p)$ |
| :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{a})$ | T | T | T | T |
| $(\mathrm{b})$ | T | F | T | T |
| $(\mathrm{c})$ | F | T | F | T |
| $(\mathrm{d})$ | F | F | T | T |

### 3.2 Contingent sentences

Sentences like $(p \rightarrow q) \rightarrow p$, parenthesized THIS way, are called contingent sentences, They are sometimes true and sometimes false.

|  | $p$ | $q$ | $p \rightarrow q$ | $(p \rightarrow q) \rightarrow p$ |
| :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{a})$ | T | T | T | T |
| $(\mathrm{b})$ | T | F | F | T |
| $(\mathrm{c})$ | F | T | T | F |
| $(\mathrm{d})$ | F | F | T | F |

Another example:

|  | $p$ | $q$ | $p \rightarrow q$ | $p \&(p \rightarrow q)$ | $\sim q$ | $(p \&(p \rightarrow q)) \rightarrow \sim q$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{a})$ | T | T | T | T | F | F |
| $(\mathrm{b})$ | T | F | F | F | T | T |
| $(\mathrm{c})$ | F | T | T | F | F | T |
| $(\mathrm{d})$ | F | F | T | F | T | T |

### 3.3 Contradictions

Some sentences have truth tables that always make them false. Such sentences are called contradictions, because they can't help but be false:

|  | $p$ | $\sim p$ | $p \& \sim p$ |
| :---: | :---: | :---: | :---: |
| $(\mathrm{a})$ | T | F | F |
| $(\mathrm{b})$ | F | T | F |

Another example:

|  | $p$ | $q$ | $p \rightarrow q$ | $\sim(p \rightarrow q)$ | $q \& \sim(p \rightarrow q)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{a})$ | T | T | T | F | F |
| $(\mathrm{b})$ | T | F | F | T | F |
| $(\mathrm{c})$ | F | T | T | F | F |
| $(\mathrm{d})$ | F | F | T | F | F |

### 3.4 Logical Equivalence

Sometimes two distinct sentences have exactly the same truth values in all circumstances. Such sentences are logically equivalent.

3．4．1 $p \rightarrow q$
3．4．2 $\sim p \vee q$
3．4．3 $\sim q \rightarrow \sim p$
3．4．4

|  | $p$ | $q$ | $\sim q$ | $\sim p$ | $p \rightarrow q$ | $\sim p \vee q$ | $\sim q \rightarrow \sim p$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(\mathrm{a})$ | T | T | F | F | T | T | T |
| $(\mathrm{b})$ | T | F | T | F | F | F | F |
| $(\mathrm{c})$ | F | T | F | T | T | T | T |
| $(\mathrm{d})$ | F | F | T | T | T | T | T |

3．4．5 $(\sim p \vee q) \Longleftrightarrow(p \rightarrow q) \Longleftrightarrow(\sim q \rightarrow \sim p)$

## 4 Predicates and Arguments

Nouns，verbs，and adjectives have predicates as their semantic values．

## 4．1 Transitive and intransitive verbs

－A simple intransitive verb example
4．1．1 John walks．
4．1．2 Walk（ j ）

4．1．1 $\llbracket$ Johnwalks $\rrbracket=$ true iff $\llbracket$ John 】 $\in \llbracket$ walks 】
4．1．2【Walk $(\mathrm{j}) \rrbracket=$ true iff $\llbracket \mathrm{j} \rrbracket \in \llbracket$ walks $\rrbracket$
－Points of interest
$-\llbracket$ Walk 】is a set，just the way 【walks 】was in our little extensional mini－semantics in Chapter 1.
－Later we will want to replace such extensional denotations with intensional denotations just the way we did in Chapter 1.

- The important thing is 4.1a). The fact that we redo this semantics in logical notation in (4.1b) is just a convenience.
- The is not to translate English into logic. The goal is to give English sentences a semantics. (4.1b) doesn't do this any better than 4.17).
- The logical notation is just a convenience for helping us get our heads clear.
- Historically the program of giving a language a clear and complete semantics comes from logic. THis is why logic has some useful ideas for us.
- Transitive verb
4.1.1 John loves Mary
4.1.2 Love ( j , m )


### 4.2 Nouns, Adjectives

- A simple nounexample
4.2.1 Fido is a dog.


### 4.2.2 $\operatorname{Dog}(\mathrm{f})$

- But what about ...?

A dog barked.
Different meaning of dog? Hopefully not!

- A simple adjectival example
4.2.1 Fido is happy.
4.2.2 Happy (f)


### 4.3 Sentential connectives retained in Predicate Logic

4.3.1 John doesn't love Mary
4.3.2 $\sim \operatorname{Love}(\mathrm{j}, \mathrm{m})$
4.3.1 John loves Mary and Fred loves Sue.
4.3.2 $\operatorname{Love}(\mathrm{j}, \mathrm{m}) \& \operatorname{Love}(\mathrm{f}, \mathrm{s})$
4.3.1 If John loves Mary, then Fred loves Sue.
4.3.2 $\operatorname{Love}(\mathrm{j}, \mathrm{m}) \rightarrow \operatorname{Love}(\mathrm{f}, \mathrm{s})$

## 5 Quantifiers

### 5.1 A; Some

- Proposal to be amended
5.1.1 John loves someone.
5.1.2 Love ( $\mathrm{j}, x$ )
- Next step in the proposal to be amended: Treat nouns and adjectives as above. Treat $a$ like some:
5.1.1 A dog is happy.
5.1.2 $\operatorname{Dog}(x) \& \operatorname{Happy}(x)$

Ditto for other predicate words
5.1.1 A dog barked.
5.1.2 Dog $(x) \& \operatorname{Bark}(x)$
5.1.1 John drives a Buick.
5.1.2 Drive ( $\mathrm{j}, x) \&$ Buick $(x)$

- We're using \& even though the word and hasn't occurred in the sentence.
- \& is going to turn out to have a lot more uses in our logical translations than just as a translation of and
- Other sentential logical connectives will also turn up in surprising places
- The problem
5.1.1 John doesn't drive a Buick.


### 5.1.2 ~[Drive $(\mathrm{j}, x) \&$ Buick $(x)]$

Does this mean the right thing?
There is an $x$ such that it's not the case that $x$ is a Buick and John drives $x$.

Capturing truth conditions: (5.1,2) gives the wrong truth-conditions for (5.1.1)
5.1.1 Suppose B11 and B12 are both Buicks. John drives B11 and John doesn't drive B12.
5.1.2 Then there is an $x$ such that it's not the case both that $x$ is a Buick and John drives $x$. Namely B12. While B12 is a Buick John doesnt drive it.
5.1.3 So the logical formula (5.1.2) comes out true in these circumstances.
5.1.4 But the English sentence (5.1.1) is not true in these circumstances. John shouldn't be driving ANY Buicks, yet he's driving B11.
5.1.5 The logical formula (5.1.2) misdescribes the truth conditions of (5.1.1) .
5.1.6 This is the semantic analogue of the grammar mis-describing the grammaticality of a sentence.

- Solution
5.1.1 John drives a Buick.
5.1.2 $\exists x[$ Drive $(\mathrm{j}, x) \wedge \operatorname{Buick}(x)]$
5.1.3 $\exists x[$ Drive $(\mathrm{j}, x) \wedge$ Buick $(x)]$ is true iff there is some entity $\mathbf{b}$ such that $[$ Drive $(j, \quad b) \wedge$ Buick (b)] is true.
5.1.4 True whenever John drives any entity that is a Buick
5.1.5 False only if there is NO entity that is a Buick that John drives
5.1.6 $\llbracket \exists x \phi(x) \rrbracket=$ true iff there is some entity $\mathbf{b}$ such that $\llbracket \phi(x) \rrbracket^{\mathbf{b} / x}=$ true
5.1.1 John doesn't drive a Buick.
5.1.2 $\sim \exists x[$ Drive $(\mathrm{j}, x) \wedge$ Buick $(x)]$
5.1.3 $\sim \exists x[$ Drive $(\mathrm{j}, x) \wedge$ Buick $(x)]$ is true iff it is not the case that there exists some entity b such that [Drive ( $\mathbf{j}$, b ) $\wedge$ Buick ( $\mathbf{b}$ )] is true.
5.1.4 Previously: $\sim[$ Drive $(\mathrm{j}, x) \wedge \operatorname{Buick}(x)]$ is true iff there exists some entity b such that it is not the case that [Drive $(\mathbf{j}, \mathbf{b}) \wedge$ Buick (b)] is true.


## Other Fixes

5.1.1 A dog is happy.
5.1.2 $\exists x[\mathbf{D o g}(x) \wedge$ Happy $(x)]$
5.1.3 A dog barked.

### 5.1.4 $\exists x[\operatorname{Dog}(x) \wedge \operatorname{Bark}(x)]$

5.1.5 Fido is a dog.
$-\exists x[\operatorname{Dog}(x) \wedge x=$ fido $]$

- Equivalent to: dog ( fido )
- Uniform treatment of a dog


### 5.2 Every and All

We use $\forall x$ to mean "for all x"
5.2.1 $\llbracket \forall x \phi(x) \rrbracket$ is true iff for every $\mathrm{x}, \phi(x)$ is true. $\phi(x)$ stands for any formula that contains x .
5.2.2 So we need to look at a large number of cases; Each needs to turn out true.
5.2.3 How many cases? All of them. Every entity in the universe.

Translating English into logic can be difficult. The meaning of the logic is rigorously defined and may not always do what youy think it does:
5.2.1 Every dog is a mammal.
5.2.2 $\forall x[\operatorname{Dog}(x) \wedge \operatorname{Mammal}(s)]$
5.2.3 This is the Wrong semantics.
5.2.4 This requires every entity in the universe to be a dog and every entity to be a mammal.
5.2.5 Paraphrase: Everything is a dog and a mammal.
5.2.6 We make no distinction between the truth condiutions of every dog is a mammal (true) and Every mammal is a dog. (false)

- Right semantics
5.2.1 Every dog is a mammal.
(a) $\forall x[\operatorname{Dog}(x) \rightarrow \operatorname{Mammal}(s)]$
(b) This says of every entity in the universe: if it's a dog, then it's a mammal.
(c) Paraphrase: Everything that is a dog is a mammal.
(d) Every mammal is a dog.
(e) $\forall x[\operatorname{Mammal}(x) \rightarrow \operatorname{Dog}(s)][$ different truth-conditions from (b)]
- Combining every and some
5.2.1 Sentences which combine every and some may be ambiguous!
5.2.2 Every man loves some woman (Reading One)
$\forall x[\operatorname{Man}(x) \rightarrow \exists y[\operatorname{Woman}(y) \wedge \operatorname{Love}(x, y)]]$
5.2.3 Every man loves some woman (Reading Two)

$$
\exists y[\operatorname{Woman}(y) \wedge \forall x[\operatorname{Man}(x) \rightarrow \operatorname{Love}(x, y)]]
$$

5.2.4 Some woman loves every man. (Reading One)

$$
\exists y[\boldsymbol{\operatorname { W o m a n }}(y) \wedge \forall x[\operatorname{Man}(x) \rightarrow \operatorname{Love}(y, x)]]
$$

5.2.5 Some woman loves every man. (Reading Two)

$$
\forall x[\operatorname{Man}(x) \rightarrow \exists y[\operatorname{Woman}(y) \wedge \operatorname{Love}(y, x)]]
$$

- Scope ambiguities: What are they?
5.2.1 Lexical ambiguity
(a) John had all the good lines.
- Reading one: line is a short segment of dialogue in a play or performance
- Reading two: line is a length of cable or rope
(b) Scope ambiguities do not involve lexical ambiguity!


### 5.2.2 Syntactic Ambiguity

(a) I shot an elephant in my pajamas

- Reading one: I was in my pajamas at the time
- Reading two: "How he got in pajamas I'll never know!" (Groucho Marx)
- Two trees
(b) Scope ambiguities do not involve syntactic ambiguity!


### 5.3 Deriving scope ambiguities

### 5.3.1 Start simple

(a) [s Every $\operatorname{man}_{x}$ walks.]
(b) $[\mathrm{S} \text { [Every man }]_{x}$ [s $x$ walks.]]
(c) $\left[\mathrm{SEvery}_{x}[x \operatorname{man}]_{x}[\mathrm{~S} x\right.$ walks. $\left.]\right]$
(d) Every $x[\operatorname{man}(x) \rightarrow$ walk $(x)]$

### 5.3.2 Two NPs

(a) $\left[\mathrm{S}[\text { Every man }]_{x}\right.$ loves $\left.[\text { some woman }]_{y}\right]$
(b) $\left.\left[\mathrm{S}[\text { Every man }]_{x}[\mathrm{~S} x \text { loves [some woman }]_{y}.\right]\right]$
(c) $\left[\mathrm{S}^{\text {Every }}{ }_{x}[x \mathrm{man}]_{x}\left[\mathrm{~S} x\right.\right.$ loves some woman $\left.\left._{y}.\right]\right]$
(d) $\left[\mathrm{S}\left[\right.\right.$ some $\left.^{\text {woman }_{y}}\right]\left[\mathrm{S}_{\mathrm{S}}\right.$ Every $_{x}[x \operatorname{man}]_{x}[\mathrm{~S} x$ loves $\left.\left.y].\right]\right]$
(e) $\left[\mathrm{S}^{\operatorname{some}}{ }_{y}\left[y\right.\right.$ woman $_{y}{ }_{[\mathrm{S}}$ Every $_{x}[x \operatorname{man}]_{x}[\mathrm{~S} x$ loves $\left.\left.y].\right]\right]$
(f) Some $y[\operatorname{woman}(y) \wedge$ Every $x[\operatorname{man}(x) \rightarrow$ love $(x, y)]]$
5.3.3 Two NPs another way
(a) $\left[\mathrm{S}[\text { Every man }]_{x}\right.$ loves $\left.[\text { some woman }]_{y}\right]$
(b) $\left[\mathrm{S}[\text { Some woman }]_{y}[\mathrm{~S} \text { [every man }]_{x}\right.$ loves $\left.\left.y.\right]\right]$
(c) $\left[\mathrm{S}_{\mathrm{Some}}^{y}\right.$ [ $[y \text { woman }]_{y}\left[\mathrm{~S}[\text { every man }]_{x}\right.$ loves $\left.\left.y.\right]\right]$
(d) $\left[\mathrm{S}\left[\right.\right.$ every $\left.\operatorname{man}_{x}\right]\left[\mathrm{S}\right.$ some $_{x}[y \text { woman }]_{x}[\mathrm{~S} x$ loves $\left.\left.y].\right]\right]$
(e) $\left[\mathrm{S}^{\text {every }}{ }_{x}[x \mathrm{man}]_{x}\left[\mathrm{~S}\right.\right.$ some $_{y}[y \text { woman }]_{x}[\mathrm{~S} x$ loves $\left.\left.y].\right]\right]$
(f) Every $x[\operatorname{man}(x) \rightarrow$ some $y[\operatorname{woman}(y) \rightarrow$ love $(x, y)]]$
5.3.4 So now this IS a syntactic ambiguity
5.3.5 A new architecture for grammar
(a) Classical aspects model (c. 1965)
(b) T-model (c. 1976)
5.3.6 Some evidence against the classical model:
(i) Everyone in this room speaks two languages.
(ii) Two languages are spoken by everyone in this room.

$$
\begin{aligned}
& 2 x \text { language }(x) \wedge \forall y \text { person }(y) \rightarrow \operatorname{speak}(y, x) \\
& \forall y \text { person }(y) \rightarrow 2 x \text { language }(x) \wedge \operatorname{speak}(y, x)
\end{aligned}
$$

(iii) It is certain that no one will leave
(iv) No one is certain to leave.

$$
\begin{aligned}
& \sim \exists x \text { person }(x) \wedge \square_{\mathrm{E}} \text { leave }(x) \\
& \square_{\mathrm{E}} \sim \exists x \text { person }(x) \wedge \text { leave }(x)
\end{aligned}
$$

## 6 Logical Entailments

A sentence A entails another sentence B if, whenever A is true, B must also be true. In this case we write:

$$
A \Rightarrow B
$$

1.Fido is a dog. $\Rightarrow$

Fido is a mammal.
2.John won the game. $\Rightarrow$

John played the game.
3.John was convicted of treason. $\Rightarrow$

Treason is a crime
4.Some man is mortal $\Rightarrow$

There exists a man.

There are different kinds of entailments. Part of the subject matter of this course is what the different kinds of entailments are. One distinction that's important is that an entailment may be true because of the laws of logic, or it may be true because of the meanings of the words involved. An entailment true because of the laws of logic is called logical entailment or a logical implication. .

Of the entailments above only 4. is a logical entailment

## 7 Relations

When we give a predicate logic analysis of an English sentence, we are breaking the sentence down in a number of independent relations. There are some linguistically significant choices being made when we do that.

When you choose a relation for a verb, it's not the syntactic order of the arguments you're trying to capture, but the roles in the relation. Be consistent, always using the same number of arguments, and keeping the same roles in the argument position in the relation.

### 7.1 Order of arguments

| Wrong | John gave the book to Mary. <br> GIVE(j,the book, m) |
| :--- | :--- |
| Right | The book was given to Mary by John. <br> GIVE(the book, m, j) |
| John gave the book to Mary. <br> GIVE(j,the book, m) |  |
| Why book was given to Mary by John. |  |
| GIVE(j,the book, m) |  |

### 7.2 Number of arguments

Consistency issues also arise when the NUMBER of arguments of a verb changes. The number of arguments of a logical predicate should always be the same. The number of arguments of a predicate is called its arity. The arity of a predicate should never change.

| Wrong | John ate the apple. <br> EAT(j,the apple) |
| :--- | :--- |
| Right <br> John ate. <br> EAT(j) |  |
| John ate the apple. <br> EAT(j,the apple) |  |
| John ate. <br> $\exists x$ EAT(j, x) |  |
|  | $\operatorname{EAT}(\mathrm{j}$, the apple) <br> $\operatorname{EAT}$ is 2-place relation |

### 7.3 Existential entailment I

Our treatment of EAT makes a prediction about the entailments of sentences with the verb eat:

$$
\begin{aligned}
& \text { John eats } \Rightarrow \\
& \text { John eats something }
\end{aligned}
$$

Because we give both sentences the same translation:

$$
\begin{array}{ll}
\text { John eats something } & \exists x \text { eat }(\mathrm{j}, x) \\
\text { John eats } & \exists x \operatorname{eat}(\mathrm{j}, x)
\end{array}
$$

This is called an existential entailment. An existential entailment is an entailment that something exists.

### 7.4 Existential Entailment II

Most English verbs have existential entailments in the following sense:

John saw Mary. $\Rightarrow$
There exists something that John saw.
In logic, the first translation entails the second:

$$
\begin{aligned}
& \operatorname{see}(\mathrm{j}, m) \\
& \exists x \operatorname{see}(\mathrm{j}, x)
\end{aligned}
$$

Another case:
John ate an apple. $\Rightarrow$
There exists an apple that John ate.
In logic, the two sentences have the same translation:
$\exists x$ eat $(\mathrm{j}, x) \wedge$ apple $(x)$
$\exists x$ eat $(\mathrm{j}, x) \wedge$ apple $(x)$
Even more simply:

$$
\text { John ate an apple } \Rightarrow
$$

There exists something that John ate.
In logic, again, the first translation entails the second:

$$
\begin{aligned}
& \exists x \text { eat }(\mathrm{j}, x) \wedge \text { apple }(x) \\
& \exists x \text { eat }(\mathrm{j}, x)
\end{aligned}
$$

In general,

$$
p \wedge q \Rightarrow p
$$

Read $\Rightarrow$ as "entails" in logic too. It is different from $\rightarrow$. Technically, $\alpha \Rightarrow \beta$ means $\alpha \rightarrow \beta$ is a tautology.

The following existential entailment also holds:

John ate an apple $\Rightarrow$
There exists something that ate an apple. $\Rightarrow$ An apple was eaten.

In logic, the first translation again entails the second:

$$
\begin{aligned}
& \exists x \text { eat }(\mathrm{j}, x) \wedge \text { apple }(x) \\
& \exists y \exists x \text { eat }(y, x) \wedge \operatorname{apple}(x)
\end{aligned}
$$

So eat has existential entailments for both its subject position and object position.

You can think of existential entailment as semantic obligatoriness. Eating can't go on without something filling both the eater and the eaten roles. But remember: existential entailment is quite different from syntactic obligatoriness:
(i) John devoured the apple.
(ii) John devoured something.
(iii) * John devoured.

The verb devour has an existential entailment on the direct object position. And, independently of that, that second argument position is obligatory.

Not every verb gives an existential entailment for every argument position:

John is looking for a unicorn $\nRightarrow$
There exists a unicorn that John is looking for.
Because of this, it's not clear how to translate John is looking for a unicorn into predicate logic. This translation

$$
\exists x \text { unicorn }(x) \wedge \text { look_for }(\mathrm{j}, x)
$$

is wrong because it immediately entails that what John is looking for exists. Under standard assumptions about what the translations mean, this sentence can't be translated into predicate-logic.

### 7.5 Arguments and entailments

On the basis of consistency, we stated that the number of arguments of a logical predicate should always be the same. But we also want to correctly represent existential entailments.

This means that SOME verbs can't be translated with one predicate.

| Wrong | Natasha kicked Boris. <br> kick $(n, b)$ <br> Natasha kicked. $\exists x \operatorname{kick}(n, x)$ |
| :---: | :---: |
| Right | Natasha kicked Boris. |
| Why | Natasha kicked. $\nRightarrow$ <br> There exists something that Natasha kicked. |


| Existential entailment |  |  |
| :--- | :--- | :---: |
| John ate | $\Rightarrow$ | John ate something. |
| John kicked | $\nRightarrow$ | John kicked something. |
| John replied | $\Rightarrow$ | John replied to something/someone. |

Another example:
(a) Fred burned the house.
(b) \# Fred burned.

Fred not filling the same role!
(c) The house was burned by Fred. Same meaning as (a)?
(d) The house was burned. Same relation as (a)?
(e) The house burned. Same relation as (a)?

Should we use the same relation in (a), (d) and (e)? This question is answered by asking if

The house was burned $\Rightarrow$
Someone/thing burned the house.
And if:

The house burned $\Rightarrow$
Someone/thing burned the house.
Here's another interesting fact. Consider purpose clauses:
John went into town (in order) to buy some bubble gum.
Purpose clauses usually require some rational entity capable of purpose in context in order to be interpreted. Consider:
(f) The house was burned to collect the insurance.
(g) \# The house burned to collect the insurance.

Other verbs like burn:

|  | English causative alternation |
| :--- | :--- |
| heat | The soup heated. <br> cool <br> John heated the soup. <br> break <br> The soup cooled. <br> John cooled the soup. <br> The vase broke. |
| move | John broke the vase. <br> The lid moved. |
| wiggle | John moved the lid. <br> John's toe wiggled. <br> John wiggled his toe (in greeting). |
| Conclusions |  |

7.5.1 The English causative alternation is productive and is distinct from object-drop in that the subject role changes!

Object drop John ate the pretzel. John is eater
John ate John is eater
Causative John broke the pretzel. John is breaker
The pretzel broke. The pretzel is eaten
Which? John turned the statue.
John turned.
Which? John cooked the eggs.
The eggs cooked (for 3 minutes).
John cooked. (unlike most of his male friends)
Which? John hammered the nail.
John hammered (away).
7.5.2 Object drop usually gives rise to an existential entailment, but some verbs do object drop with no existential entailment (kick).
7.5.3 The intransitive verbs in the causative alternation generally do NOT have an existential entailment:

The house burned. $\nRightarrow$
Someone/thing burned the house.
7.5.4 Passives generally have an existential entailment:

John ate the pretzels.

The pretzels were eaten. $\Rightarrow$
Someone ate the pretzels.

### 7.6 Oblique Arguments

7.6.1 An oblique argument is an argument of a relation that is marked with a preposition. (Syntactically, oblique arguments aren't direct arguments; that is, they aren't subjects or direct objects or second objects).
7.6.2 An oblique argument almost always carries an existential entailment:

John replied $\Rightarrow$
There is something/someone that John replied to.
7.6.3

| [ To whom ] did John reply $t$ ? |  |
| :--- | :--- |
| [ Her emails ] were always replied to $t$ at once ? | To whom moves as a unit. |
|  | Her emails passivizes like a <br> Direct Object. |

7.6.4 Very often: Verb + Preposition behaves like one relation: reply_to
reply_to (j , m )

We will call these argument-marking prepositions
7.6.5 Other cases (argument marking preposition in bold)

| Example |  | Relation-name | ヨ-ent. |
| :--- | :--- | :--- | :--- |
| (a) John applied (for the job). | apply_for | yes |  |
| (b) John relied $*$ (on Mary). | rely_on | yes |  |
| (c) John gave the book $*$ (to Mary.) | give (tradition!) | yes |  |
| (d) John sent the book (to Mary.) | send | yes |  |

As these examples show, if a PP is obligatory, that is a pretty good sign the preposition is argument-marking.

### 7.6.6 Now consider sit

(a) John sat under the table.
(b) John sat in the chair.
(c) John sat in the hall.
(d) John sat on the couch.
(e) John sat on the table.

If these sentences have one relation each, it's got to be a different one in each sentence.
But to say that would be to miss that there is something common going on in each. Our clue is the existential entailment:


What is entailed is that there is something supporting John's seated position. Hence the basic relation seems to be sit_on:

$$
\text { sit_on ( } \mathrm{j}, \text { the table })
$$

But what about the others?

John is sitting under the tree.
$\exists x$ sit_on $(\mathrm{j}, x) \wedge$ under $(x$, the tree $)$

### 7.7 Copular sentences

7.7.1 The copula is the verb to be. Sentences whose main verb is a form of the verb to be are called copular sentences:

Type
(a) Fido is a dog. Copular
(b) Fido is happy. Copular
(c) Fido is under the table. Copular
(d) Fido is barking. Not Copular [Main Verb = bark]
7.7.2 The phrases following the copula are called predicative phrases
(a) Fido is a dog.
Predicative NP
(b) Fido is happy.
Predicative AdjP
(c) Fido is under the table. Predicative PP
7.7.3 In the logical translations of copular sentences, we assume be contributes nothing to the meaning:
(a) $\operatorname{dog}$ (fido)
(b) happy (fido)
(c) under (fido, the table )

Thus concludes the grammar of Fido!

### 7.8 Predicative Prepositions

7.8.1 In the previous section we introduced the idea that predicative prepositions should be treated as 2-place relations:
under ( fido, the table )
7.8.2 In contrast, argument-marking prepositions are incorporated into the meaning of the verb:

$$
\begin{aligned}
& \text { rely_on ( fido, his master ) } \\
& \text { sit_on ( fido, his mat ) }
\end{aligned}
$$

7.8.3 One big advantage of treating predicative preps as 2-place relations is that the same meaning works for PPs modifying nouns:
(a) A dog under the table barked.
(b) $\exists x \operatorname{dog}(x) \wedge$ under $(x$, the table $) \wedge \operatorname{bark}(x)$
(c) A traveler from Spain arrived.
(d) $\exists x$ traveler $(x) \wedge$ from $(x$, Spain $) \wedge$ arrive $(x)$

### 7.9 Places and paths

7.9.4 The same two-place preposition meaning introduced in the last section also works for predicative PPs that do not follow the copula:
(a) John put a vase under the table.
(b) John put a vase on the table.
(c) John put a mirror behind the flowers.

These PPs are called predicative as well because they also seem to select a 2-place relation; the fact that so many prepositions are appropriate for the same syntactic slot supports the idea that their main communicative function is to distinguish among distinct 2-place spatial relations. Also, if they were argument-marking PPs we would have
to have multiple puts: put_under, put_on, put_behind. But in fact there just seems to be one put which means something like CAUSE то CHANGE LOCATION.
Other verbs with the same properties; place, hide, slip, insert, and throw. In all such cases I will refer to the thing changing locations as the theme and to the location it arrives at as the goal. Other verbs of change-of-location?
The semantics we will assume for such examples is:
(a) John put a book on the table
(b) $\exists x, y \operatorname{book}(x) \wedge \operatorname{put}(\mathrm{j}, x, y) \wedge$ on $(y$, the table $)$
(a) John put a book under the table
(d) $\exists x, y \operatorname{book}(x) \wedge \operatorname{put}(\mathrm{j}, x, y) \wedge$ under $(y$, the table $)$

Here $y$ is the goal, an implicit syntactically unexpressed place, the place where John put the book. That place in (a) is on the table, and in (c) under the table.
7.9.5 Sometimes more than one PP modifies the goal:
(a) John hid the flowers under the table behind the vase.
(b) $\exists x, y \operatorname{book}(x) \wedge \operatorname{put}(\mathrm{j}, x, y) \wedge$ under $(y$, the table $) \wedge$ behind $(y$, the vase $)$

In this case there is a reading on which the place the flowers end up is both under the table and behind the vase. This is the reading represented in (b).
7.9.6 The same multiple PP effects can be observed with path PPs, which describe the trajectory of the theme of a motion verb:

The sled glided over the river through the woods to grandmother's house.

In this case there's a path with an implicit order that can be made explicit with preps like from and to. More about these later.

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