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Exercise 1

(1) a. Oliver and Richard are roundheads.
   b. Oliver and Richard are relatives.
   c. Oliver and Richard like to drink to each other.

Paraphrasing the sentences in (1) as conjoined sentences $p$ and $q$, as in (2), works for (a) and (c), and fails in the case of (b):

(2) a. $p = \text{Oliver is a roundhead.}$
    $q = \text{Richard is roundhead.}$
    $p \& q$

b. $p = \text{Oliver is a relative.}$
   $q = \text{Richard is a relative.}$
   $p \& q$

c. $p = \text{Oliver likes to drink}$
   $q = \text{Richard likes to drink.}$
   $p \& q$
Exercise Two

a. $p \rightarrow q$ = If this is summer, it’s damned cold.
   $p$ = this is summer
   $q$ = it’s damned cold.

b. $p \land q$ = Lemons look good, but taste sour.
   $p$ = Lemons look good.
   $q$ = Lemons taste sour.

c. $q \rightarrow p$ = You can if you want to.
   $p$ = You can do x
   $q$ = You want to do x.

d. $(p \lor q) \land \neg r$ = He will come today or tomorrow but not later.
   $p$ = He will come today
   $q$ = He will come tomorrow
   $r$ = He will come later than tomorrow
   $r$ ≠ He will not come later than tomorrow
Exercise Two, ctd.

e. \( \sim (p \lor q) \rightarrow r \) = If neither God nor the devil exists, it’s difficult to be religious.
\( \sim (p \land q) \rightarrow r \) ≠ If neither God nor the devil exists, it’s difficult to be religious.

\( p \) = God exists.
\( q \) = The Devil exists
\( r \) = It’s difficult to be religious.

f. \( p \lor q \) = Throw the cat out or I will leave.
\( \sim p \rightarrow q \) = Throw the cat out or I will leave.
\( p \) = [You] throw the cat out.
\( q \) = I will leave.
Exercise 3

a. \[ \text{Neg}(f) \]
\[ \sim \quad \text{p}(t) \]

b. \[ \sim \quad \text{Conj}(f) \]
\[ \text{p}(t) \quad \& \quad \text{r}(f) \]

c. \[ \sim \quad \text{Disj}(t) \]
\[ \text{p}(t) \quad \lor \quad \text{r}(f) \]
Exercise 3, ctd.

d.

```
Disj(t)
/   \  
p(t)  v  Conj(f)
|      |    
|      |    q(t) & r(f)
```

e.

```
Impl(t)
/   \  
r(f)  -->
|      |
|      |      Disj(t)
|      |    /   \  
|      |    Conj(f)  v  Disj(t)
|      |      /      
|      |     q(t) & r(f)  p(t)  v  q(t)
```
Exercise 3, ctd.

\[
\text{f.}
\]

\[
\text{Eq}(t)
\]

\[
\text{r}(f) \leftrightarrow \text{Conj}(f)
\]

\[
\text{p}(t) \& \text{r}(f)
\]
# Exercise 4a: Tautologies

\[ \sim (p \& \sim p) \]

<table>
<thead>
<tr>
<th></th>
<th>\sim p</th>
<th>p &amp; \sim p</th>
<th>\sim (p &amp; \sim p)</th>
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Is a tautology!
Exercise 4b: Tautologies

\[(p \lor q) \rightarrow p\]

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Is not a tautology!

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Exercise 4c: Tautologies

\[ \sim (p \& q) \equiv (\sim p \lor \sim q) \]

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<tr>
<th>p</th>
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<th>\sim (p &amp; q)</th>
<th>\sim p</th>
<th>\sim q</th>
<th>\sim p \lor \sim q</th>
<th>\sim (p &amp; q) \equiv (\sim p \lor \sim q)</th>
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</thead>
<tbody>
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Is a tautology!

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<th>p</th>
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Is the conjunction *because* truth-functional?

<table>
<thead>
<tr>
<th></th>
<th>George Bush won the election of 2000</th>
<th>because</th>
<th>Al Gore failed to win key swing states.</th>
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<tr>
<td>F</td>
<td>George Bush won the election of 2000</td>
<td>because</td>
<td>The Rams won the superbowl.</td>
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Not truth functional because the truth of the complex sentence is not a function of the truth of its constituent sentences.