Paths and Scalar Change

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1 Introduction

Consider the sentences in (1):

(1) a. The fog extended (from the pier to the point).
b. The crack widened (from the north tower to the gate.)
c. The storm front zigzagged (through the entire state of Colorado)
d. Snow covered the mountain (from the valley floor to the summit).

Each of sentences (1a)-(1d) has both an event reading and a stative reading. For example, on what I'll call the *event reading* of sentence (1a), a body of fog beginning in the vicinity of the pier moves pointwards, and on the other, stative reading, which I'll call an *extent* reading, the mass of fog sits over the entire region between pier and point. The event reading entails movement. The extent reading entails extension, the occupation of a region of space. Similarly, there is a reading of (1b) describing a crack-widening event, as well as a reading describing the dimensions of the crack, increasing in width along an axis extending from the north tower to the gate; and readings of (c) and (d) describing movement events as well as readings describing the configuration of the storm front and the snow respectively.

Building on the Hay et al. (1999), Gawron (2006) proposes an analysis of the first 3 cases assuming the lexical semantics of each predicate includes a *state function*, a function from indices to a state space. In the case of *widen* this is a function to degrees, and in the case of *extend*, a function to locations. The distinction between event and extent readings depends on whether the domain of the function is the time axis (event reading) or some contextually provided spatial axis (extent readings). Call this the GHKL analysis. The central claim of the GHKL analysis is that extent readings can still describe change, although change along a spatial axis. Thus extent verbs may be verbs of change, not just simple statives, and may show aspectual properties such as gradual change and telicity along a spatial axis. For example, there are both spatial accomplishments and spatial activities, as shown in (2).

(2) a. The crack widened nearly half an inch in ten meters.
b. The crack widened for 100 yards.

The fact noted by Jackendoff (1990), that predicates exhibiting this ambiguity take path-phrase modifiers on both readings, is due to the fact that **path** is the only semantic component available to introduce the spatial axis, and path-phrases the only way to describe orientation of the axis.

Missing from Gawron (2006) is an articulation of how extent predicates fit in with a general account of state functions and telicity such as the one outlined in Kennedy and Levin (2001). This paper seeks to identify the specific properties making a predicate an extent predicate, locate them with respect to other verbs of gradual change, and to account for some of the variation in the aspectual nature of extent predicates, including the important case of cover/fill verbs as in (1d), unanalyzed in Gawron (2006).

Two diagnostics of aspectual structure will be examined. First consider the compatibility of the adverb *gradually* with extent and event readings:

(3) Graduality

[- $Grad_X$]	(a)	The fog gradually covered the peninsula
	(b)	The fog gradually extended to the point.
[+ $Grad_X$]	(c)	The crack gradually widened from the tower on.
	(d)	The storm front gradually zigzagged to the border.

All the predicates in (3) are compatible with the adverb *gradually* on at least one reading. The sentences marked [- $Grad_X$] have only event readings; sentences marked [+ $Grad_X$] have both event and extent readings.

Given the structure of the GHKL analysis, it is fairly significant that there are cases in which *gradually* is compatible with event readings but not the corresponding extent readings. Assume a fairly natural account of *gradually* like that of Pinon (2000): graduality requires that a degreeable state function be increasing.[1] If the only difference between an event reading and a state reading is the domain of the state function, why is the state function increasing in one case and not increasing in the other?

There is another property that correlates with the feature [+ Grad_X] which is of relevance. The [+ Grad_X] predicates are degreeable; the [- Grad_X] predicates may not be:

(4) a. The road widened sharply.
b. The road zigzagged sharply.
c. # The shadows covered the patio sharply.
d. # The shadows extended sharply.

Although it can be difficult to determine what adverbials modifiers are truly degree modifiers, and therefore which are diagnostics of degreeable predicates, I will argue that the predicates in (4a) and (4b) exhibit what I call **change by degrees** and that the predicates in (4c) and (4d) exhibit what I call **change by parts**. That is, in one case the state functions return degrees on a scale, in the other, parts in a mereology.

On the basis of this observation, one might be tempted to try to explain the the graduality facts in (3) simply by saying that *gradually* requires change by degrees, but this does not account for the fact that the non-degreeable predicates in (3) *do* combine with *gradually* on the event reading. I will argue that the relevant constraint on *gradually* is that it requires change. Thus, *gradually* is compatible with change by parts; the problem with the extent readings of (3a) and (3b) is that they use spatially indexed state functions, and spatially indexed state functions can describe change only when they map to degrees. In contrast, temporally indexed state functions can describe both change by parts and change by degrees, because temporally indexed state functions have ranges with a different structure.

Extent predicates share the property that they all take paths. A second dimension of aspectual variation is whether path is really an incremental theme (Dowty 1991) in event readings; that is, do the truth conditions require that the path covered grow homomorphically with the event, with the location identified in the *from* phrase overlapped at the beginning of the event, and the location identified in the *to*-phrase overlapped at the end?

(5) Incrementality

[+ Incr_e]	(a)	A storm front zigzagged from Prescott to the border.
	(b)	The fog extended from the pier to the point.
[- Incr_e]	(c)	The crack widened from the tower to the north gate.
	(d)	Fog covered the peninsula from the pier to the point

For the cases marked [- $Incr_e$], the answer is no. In particular, on the non-incremental event reading of (c) the progression of the crack's widening may be in any order, say, from gate to tower, as long as the event concludes with a widening that covers that span; and in (d) the fog's progress may be in any order as long as in the end a span between pier and point is covered.

Accounting for these differences in aspectual nature will lead us to posit two kinds of semantic differences among extent predicates. First, some will be spatial accomplishments and activities; and some, spatial states. Second, there will be two kinds of change involved, change by parts and change by degrees, and the differences in behavior of these two kinds of change will crictical to capturing the graduality facts. This provides considerable motivation for allowing state functions for verbs of gradual change that take their range in what I'll call a **mereology**. Mereologies in the sense used here would include scales with degrees, as well as domains such as quantities of stuff, locations, groups, and paths.

The account relies on the idea that axes of change may be added to predicates (through the use of an INCREASE operator). Predicting the actually occurring possibilities requires the fairly natural assumption that predicate may have only one axis of change. A somewhat revised but fairly intuitive notion of stative predicate can now be recovered through the notion of a 0-dimensional predicate, a predicate with no dimension of change (Jackendoff 1996).

The plan of this paper is the following:

(1) Introduction (this section)
(2) Basic analysis:
  (2.1) Change and spatial dimensions
  (2.2) Paths: Axes of reference and axes of change
(3) Aspectual Variation
  (3.1) Graduality
  (3.2) incrementality
(4) Motivating aspectual variation: 0 and 1 dimensional predicates in lexical structure
(5) Mereologies and scales in the account of change
(6) Conclusion

2 Basic analysis

The verbs in (1) are examples of a large class of verbs exhibiting event/extent ambiguities. These verbs include EXTENT verbs discussed in (Jackendoff 1990), for example *cover, extend* , and *surround*, as well as the verbs called *path-shape* verbs in FrameNet (Fillmore and Baker 2000), listed in (6):

(6) angle, bear, bend, climb, crest, crisscross, cross, curl, descend, dip, dive, drop, edge, emerge, enter, exit, leave, meander, mount, plummet, reach, rise, round, skirt, slant, snake, swerve, swing, traverse, undulate, veer, weave, wind, zigzag

As the name suggests, the unifying semantic characteristic of path-shape verbs is that they specify the shape of a path. Either the shape is the configuration of the theme in space or the theme is moving and the verb specifies the shape of the path of motion:

(7) a. The road zigzagged up the hill. [Extent reading]
b. The halfback zigzagged to the goal line. [Event reading]

Criterial for the class is that, on extent readings, they allow inanimate paths that are extended in space in the required configuration. This distinguishes them from manner of motion verbs. Repeating some examples of from FrameNet:

(8) a. The road snaked up the hill . [path-shape]
b. # The road slithered up the hill. [manner of motion]

I will use the term **extent predicates** to describe all path-shape verbs and a few verbs from Jackendoff's list that are not path-shape verbs but which show event/extent ambiguities, such as *surround* and *extend*.

The second class of verbs that show event/extent ambiguities is a large class of degree-achievement verbs, including *narrow, warm, cool, rise, fall, darken, lengthen, lighten, brighten, dim, grow*, and all color adjectives. I will call these **extent degree achievements**. I assume that *cover* and *fill* fall in this class as well. The distinction between these two classes is provisional and descriptive. As we shall see in Section 4, these do not identify natural semantic classes.

2.1 Degree achievements

Let us begin with the analysis of the degree achievements. What is going on in the event reading of (1b)? Widths may vary in time; and events of widening in time are events in which the width of the theme at the beginning of the event differs from the width at the end. What is going on in the extent reading of (1b)? A natural idea is that (1b) exploits a contextually provided spatial axis to measure out change. Path expressions select an interval on that axis. Thus we find if we measure the width of the crack moving up along that axis in the selected interval that it is increasing. What does it mean to measure width "up along" a spatial axis? It means the points on the axis are ordered and as we moved in the "upward" direction on the axis, the width increases. What does it mean to measure the width of an object x "at a point" s on an axis? It means we imagine a plane P perpendicular to the axis and measure the width of the intersection of P with x. This means that we can have a single function

$$\mathrm{wide}(\sigma)(i)$$

that returns widths for the figure of state σ for an index i whether i is spatial or temporal. For clarity we will refer this function as $\mathrm{wide}_{\mathrm{T}}$ when the arguments are temporal and $\mathrm{wide}_{\mathrm{S}}$ when the arguments are spatial. I will reserve $\mathrm{wide}_{\mathrm{I}}$ (I for "index") to schematize over both cases).

The idea of a contextually provided spatial axis is of course already necessary to deal with other phenomena in the language of space

(9) a. The ball is behind the chair.
    b. The ball is behind the rock.

These two examples differ in that chairs have canonical backs and fronts and rocks do not. Thus a natural interpretation of (9a) exploits the canonical back-to-front axis of chairs and determines behindness accordingly. In contrast, rocks have no canonical back to front axis and the axis determining behindness must be contextually provided.

The adjective *wide* is itself an adjective that may require contextual measurement axes:

(10) a. The cabinet is 6 feet wide.
     b. The boulder is 6 feet wide

Here the cabinet has canonical orientation axes, with one usually favored for widths, but the boulder does not. It must be context, "point of view", that orients the axis. The axis along which widths are measured, called the measurement axis, must be perpendicular to the front-to-back axis in (1b), along which widths may change. I will refer to the front-to-back axis as the **axis of reference**. The essential claim of the analysis is that in the extent readings of sentences like (1b) the axis of reference may be exploited as an **axis of change**.[2] We need to distinguish the following two possibilities:

(11)

(a)	$\mathrm{wide}^{\mathrm{S}}_{\mathrm{T}}$	state function with temporal domain and ref. ax. S
(b)	$\mathrm{wide}^{\mathrm{S}}_{\mathrm{S}}$	state function with spatial domain and ref. ax. S

These alternatives are not quite symmetric. With a temporal state function, any spatial axis through the figure is in principle a possible axis of reference; with a spatial state function using S as the axis of change, only S may be the axis of reference.[3] We thus write:

$$\mathrm{wide}_{\mathrm{S}}$$

for $\mathrm{wide}^{\mathrm{S}}_{\mathrm{S}}$.

Given this picture, the analysis of degree achievements from HKL can just be carried over straightforwardly. We assume the semantics of a simple adjectival use of *wide* is:

(12) a. The crack is a half inch wide.
b. $\exists\sigma[\mathrm{wide}^{\mathrm{S}}_{\mathrm{T}}(\sigma)(t)\text{=}[.5\,\mathrm{in}] \wedge \text{figure}(\sigma)\text{=}c\,]$

Here t is some contextually provided moment of time. The subscript T tells us this is a use of *wide* as a function of times (not locations), and the superscript S denotes the spatial reference axis.

In the rest of this section we sketch the basic analysis for the following simple case:

(13) The crack widened half an inch.

This example has no path phrases. In the next section, after presenting the analysis of path phrases, we extend the basic analysis to uses of *widen* with path phrases.

To begin with, here is a slightly modified version of the axiom HKL use to define increase:[4]

(14)
$$\forall\, t_1,\, t_2,\, x,\, d \left[\begin{array}{l} \exists e\,[\ \mathrm{increase}(\mathrm{wide}^{\mathrm{S}}_{\mathrm{T}})(e)\text{=}d \wedge \\ \qquad \textsc{start}(e)\text{=}t_1 \wedge \textsc{end}(e)\text{=}t_2 \wedge \text{figure}(e)\text{=}x\,] \\ \qquad\qquad \longleftrightarrow \\ \exists\sigma_1,\, \sigma_2[\ \textsc{start}(\sigma_1)\text{=}t_1 \wedge \textsc{end}(\sigma_2)\text{=}t_2 \wedge \\ \qquad \text{figure}(\sigma_1) = \text{figure}(\sigma_2) = x \wedge \\ \qquad \text{wide}^{\mathrm{S}}_{\mathrm{T}}(\sigma_1)(t_2) = \text{wide}^{\mathrm{S}}_{\mathrm{T}}(\sigma_2)(t_1) + d\,] \end{array}\right.$$

A widening event is one that relates to two width states, the width state of the figure at the event's beginning and the width state of the figure at the end, with the difference in width measures, d, equaling the width increase of e:

$$\mathrm{increase}(\text{wide}^{\mathrm{S}}_{\mathrm{T}})(e) = d$$

The revision required to admit extent readings is quite simple: Make *increase*, START, and END all sensitive to what axis change is being measured on. Using I for the axis of change, whether temporal or spatial, and $i_1,\ i_2$ for indices on the axis, we would substitute the following into (14):

$$\ldots \text{increase}_{\mathrm{I}}(\text{wide}^{\mathrm{S}}_{\mathrm{I}})(e) = d \ldots$$
$$\ldots \text{START}_{\mathrm{I}}(e)\text{=}i_1 \wedge \text{END}_{\mathrm{I}}(e)\text{=}i_2 \ldots$$
$$\longleftrightarrow$$
$$\ldots \text{START}_{\mathrm{I}}(\sigma_1)\text{=}i_1 \wedge \text{END}_{\mathrm{I}}(\sigma_2)\text{=}i_2 \ldots$$

We call I, the axis the increase operator exploits, **the axis of change**. When I is spatial it must be a contextually supplied axis, and the most salient one is the adjectival axis of reference L, each index of which determines a cross-section of the figure with a (potentially different) width. When I is temporal, we simply have the case of (14) again.

The definition of the START of an event with respect to an axis is:[5]

(15) $$\text{START}_{\mathrm{I}}(e) \;=\; \underset{p \in \mathcal{T}(e)}{\mathrm{Min}}\ \mathrm{coordinate}(I,\, p)$$

where $\mathrm{coordinate}(I,\, p)$ is the coordinate of point p along axis I. Thus, the start and end of e along axis I are the respective minima and maxima of the projection of e's spatiotemporal trace, $\mathcal{T}(e)$, onto I. An event will thus have different starts and ends, depending on what axis is used.

We have now said enough to address the case of (13):

(16) a. $\exists\, e\ [\text{increase}_{\mathrm{S}}(\mathrm{wide}_{\mathrm{S}})(e)\text{=}[.5\,\mathrm{in}] \wedge \text{figure}(e)\text{=}c\ ]$

b. $\exists\, e\ [\text{increase}_{\mathrm{T}}(\mathrm{wide}_{\mathrm{T}})(e)\text{=}[.5\,\mathrm{in}] \wedge \text{figure}(e)\text{=}c\ ]$

The extent reading of (13) is represented in (16a) as the choice of S, the axis of reference, as the axis of change subscripting *increase*; the event reading in (16b) as the choice of T, time, as the axis of change. According to our revised version of (14), both readings true if and only if the difference in the value of the width function between the start and end of e as measured the axis of change S is .5 inch.

Note that this analysis of the ambiguity of (13) makes no use of an aspect-changing operator, such as the inchoative operator used in the analysis of extent predicates in Jackendoff (1990), to distinguish the readings. Essentially the same meaning is claimed to yield both readings, the difference residing in which axis is used for the evaluation of change. This makes the prediction that the extent readings and event readings for *widen* have essentially identical aspectual properties. That this appears to be correct is argued for by the data in (2), in which extent readings of *widen* can describe both spatial activities and spatial accomplishments. Paralleling this for event readings, *widen* falls into a sizable class of degree-achievement verbs that can be both activities and accomplishments, depending on the exact width property at issue. Example (17) illustrates this.

(17) a. The crack widened five inches in five minutes.
b. The crack widened for several hours.

Thus, the extent readings for *widen* in (2) preserve exactly the aspectual properties of the event readings. This is entirely in line with the HKL theory: Telicity should be determined only by the semantic properties of the degree, and the semantics of the degree in (2) and (17) are unchanged on this analysis. As we shall see below, however, not all extent/event ambiguities are aspect-preserving in this sense.

2.2 Paths and extent readings

We begin by defining an operator $\mathbf{path}$ which, for each appropriate event, will return the state function that tracks the location of the event's *theme* with respect to either space or time. The path operator will serve two functions:

(a) account for the use of path phrases with motion predicates, what is usually thought of as the basic sense of path phrase like *from Boston* and *to the ridge*;

(b) account for the use of path phrases in extent readings like those of (1);

2.2.1 Path operator and events

The $\mathbf{path}$ operator returns a function for each event. The function returned will be called the path of the event.[6]

We assume that

$$\mathbf{path}(e)$$

denotes the path function associated with event e, if e is an event of the appropriate type. We call **path** the path-role and $\mathrm{path}(e)$ the path-function, or more simply the path, for e.

The key property of a path for our purposes is that paths are always defined relative to an axis.

$$\mathbf{path}_{\mathrm{T}}(e)(t)$$

is the location of the theme at time t.

$$\mathbf{path}_{\mathrm{S}}(e)(s)$$

is the location of the slice of the theme that intersects the plane through axis S at s.

For any path function π, whether temporal or spatial, The domain is that set of points on the axis I that fall within e:[7]

$$\mathbf{path}_{\mathrm{I}}(e){=}\pi \quad \text{only if} \quad \pi : [\textsc{start}_{\mathrm{I}}(e),\ \textsc{end}_{\mathrm{I}}(e)] \rightarrow \mathrm{Locations}$$

Thus, there are many path functions for any given event, corresponding to the starts and ends determined by each axis through it.

Path functions always return locations at indices: Temporal path functions return regions of space, typically 3-dimensional regions; spatial path functions return slices. As we shall see this will be sufficient to predict certain structural differences between temporal and spatial paths.

2.2.2 Path-property verbs

The path operator can be directly applied to Jackendoff's extent verbs and the larger class of path-shape verbs, all of which involve motion on their event readings. For example, consider *extend* and *zigzag*:

(18) **Path property verbs**

a. *extend*: $\mathrm{extend}_{\mathrm{S}}(e) = \pi$ iff $[\mathrm{path}_{\mathrm{S}}(e) = \pi]$

$\mathrm{extend}_{\mathrm{T}}(e) = l$ iff $[\textsc{increase}_{\mathrm{T}}(\mathrm{path}_{\mathrm{T}})(e) = l]$

b. *zigzag*: $\mathrm{zigzag}_{\mathrm{I}}(e) = d$ iff $\textsc{zigzaggy}\circ\mathbf{path}_{\mathrm{I}}(e) = d$

Note that *extend* and *zigzag* are defined in terms of properties of paths, one degreeable, one not; We call all such verbs **path-property verbs** . They include all the path-shape verbs.

The treatments of *extend* and *zigzag* contrast in several respects. For one thing, an aspectual contrast has been posited between event and extent readings for *extend*, but not for *zigzag*. We take these cases in turn.

The first predicate in (18a) is intended to capture extent readings with *extend*. It introduces a spatial path, path_S, into the described state σ, which requires that the the theme of σ be extended along the path of σ.[8]

(19) a. The fog_f extended from the valley floor_v to the ridge_r

b. $\exists\sigma[\text{extend}_\text{S}(\sigma) = \pi \wedge \text{theme}(\sigma){=}f \wedge [v : r](\pi)]$

c. $\exists\sigma[\text{INCREASE}_\text{T}(\text{extend}_\text{T})(\sigma) = l \wedge \text{theme}(\sigma){=}f \wedge [v : r](\text{path}_\text{T}(e))]$

The semantics in (19b) illustrates an extent reading; (19c), an event reading. The use of the INCREASE operator in (c) may be somewhat surprising. Path is a function that returns locations. What does it mean for locations to be *increasing* and what does it mean for the location of the *theme* in an event e to increase by an amount l?

We assume that

$$l_1 \sqsubseteq l_2$$

if and only if l_1 is a subregion of l_2. Thus we assume a **partial** ordering on regions; two regions are ordered if and only if one is a part of another. Using the terminology of mereological accounts (discussed at length in Section 5) we call this a **part-of** relation. Schematically, INCREASE is defined as follows:

$$\begin{aligned}\text{increase}(\alpha)(e) = d \ \text{ iff } \exists\sigma_1\,\sigma_2\ & \alpha(\sigma_1) = d_1 \ \wedge\ \text{START}(\sigma_1) = \text{START}(e) \wedge \\ & \alpha(\sigma_2) = d_2 \ \wedge\ \text{END}(\sigma_2) = \text{END}(e) \wedge \\ & d_1 + d = d_2\end{aligned}$$

So to extend this to regions, we need to define a unique d that will function as the difference argument d.

This can be done via the **relative complement** of two regions l_1 and l_2, written l_1/l_2

$$l_1/l_2 = \underset{l}{\text{argmax}}[\, l \sqsubseteq l_1 \wedge \neg l \otimes l_2\,]$$

where $\otimes$ is the overlap relation,[9] which holds between two regions l_i and l_j when there is some region that is part of both. Then:

$$d_1 + d = d_2 \text{ iff } d = d_1/d_2$$

The use of the INCREASE operator in (18a) captures a basic descriptive fact: The motion in event readings of *extend* is spreading as opposed to incremental.

I use the term *spreading motion* to describe motion that fills an expanding region of space, and *incremental motion* to describe motion in which each region of space is vacated as the next is occupied. On an event reading of *A extended from B to C*, spreading motion is involved, because A remains in contact with B throughout the event. Note in particular that we don't get the right truth conditions if we measure change in a totally ordered domain like volume. Measuring change with volume is correct for a verb like *inflate*, where the truth conditions really do require an increase in a scalar measure, but *extend* incorporates the additional requirement that the spatial region occupied at the end of the event include the region occupied at the beginning, and this is exactly what the sub-region relation captures. [10]

The definition in (18b) says that an event is a zigzag event along axis I if and only if there is a π such that π is the path of e and π is zigzaggy to some degree d. I leave the exact definition of zigzaggyness unspecified, but it will be some function of the number of undifferentiable points and sign changes in the slope per interval.[11] The concept of zigzaggyness is a constraint on the relationship of the path to the axis of change that cross-cuts extent and event readings. Again, as with *widen* event and extent readings differ only in what axis is the axis of change. Sentence (20) provides an example:

(20) a. Mist$_m$ zigzagged from the valley floor$_v$ to the ridge$_r$
b. $\exists e,\ d$[zigzag$_\mathrm{S}(e)$=d $\wedge$ theme(e)=m $\wedge$ $[v:r] \circ$ $\mathbf{path}_\mathrm{S}(e)$]

The definition of *zigzag* differs from the definition of *extend* in introducing a degreeable function *zizaggy*. The primary motivation for this is contrasts like the following:

(21) a. The road zizagged/?extended sharply/gently up the hill.
b. The 4x4 zigzagged sharply/gently up the hill.

As we shall see this difference will be crucial in capturing the differences in graduality between *zigzag* and *extend*. Note that degree sensitive adverbs co-occur with other path-shape verbs as well:

(22) a. The road curved/rose sharply up the hill.
b. The road climbed steeply.

Indeed, some members of the class *rise*, *ascend*, and *climb* are often analyzed as degree achievements related to some adjective like *high*. What I am basically suggesting is that they are all degreeable predicates.

Thus, *extend* contrasts with both degree acheivements like *widen* and incremental motion predicates like *zigzag* in not introducing a degree in its semantics. It is like *widen*, however, in that its event reading is captured by combining a function of time with the INCREASE operator. The key intuition of the INCREASE operator analysis is that it takes a function of time — call it Δ_t — and returns a function of events — call it Δ_e — that measures the overall change in Δ_t in e. Call Δ_t the **state-function** of e and Δ_e the **change function** of e. Then according to (18a), the state-function for an *extend*-event is $\mathrm{path}_{\mathrm{T}}$, and that state-function must be increasing in an extending event. And it makes sense to talk about increase with regions and increase with degrees because both are ordered domains. Thus the difference between the predicate introducing a degree and the predicate not introducing a degree is not that great. Both incorporate state functions that can capture the amount of change in an event, because both take their domains in ordered ranges. Nevertheless, it will rpove useful to distinguish betweeen these two kinds of change. We will call change measured in a domain of degrees **change by degrees** and change measured by a partial ordering **change by parts** (because all the partial orderings we need will intuively be part-of relations). Change by parts will be revisited when we treat *cover* in Section 3.1.

The case of *zigzag* differs in one respect from both degree achievements and *extend*. There is no Δ_t; that is, no underlying stative predicate is assumed in its semantics. Indeed, it is not clear what stative predicate would produce the right semantics for an event reading, since the temporal property does not seem to be reducible to a property that can be true at an instant of time. Thus there is no INCREASE operator in the semantics of zigzagging, merely a function which returns the degree of zigzaggyness of the path of the entire event:

Predicate	Δ_t	Δ_e
extend	$\mathrm{path}_{\mathrm{T}}$	$\textsc{increase}_{\mathrm{T}}(\mathrm{path}_{\mathrm{T}})$
widen	$\mathrm{wide}^{\mathrm{S}}_{\mathrm{T}}$	$\textsc{increase}_{\mathrm{T}}(\mathrm{wide}^{\mathrm{S}}_{\mathrm{T}})$
zigzag	NA	$\textsc{zigzaggy} \circ \mathbf{path}_{\mathrm{I}}$

This predicts that *zigzag* on its event readings is not restricted to spreading motion, which is correct, as shown in (7).

Despite the absence of an INCREASE operator, note that *zigzag* can be just as much of a spatial accomplishment as *widen* can:

(23) The roads zigzagged quite a bit in just 1000 meters.

The semantics in (18b) is consistent with this fact. It is consequence of the meaning of the degreeable predicate *zigzaggy* that a zigzagging event will exhibit change. Thus, we are not proposing a uniform decomposition of accomplishments and it does not appear such a decomposition is possible, contra Dowty (1979). We return to the aspectual nature of *zigzag* in the next two sections.

2.2.3 Paths for width predicates

We now address the question of how the definition of path interacts with the analysis of *widen* sketched in the previous section.

First, note that path phrases occur with the adjective *wide* as well as with the degree achievement verb:

(24) The canyon was six feet wide from the North End_{n} to the trail head_{t}.

Since we have assumed that *wide* denotes a function evaluable either *at a moment in time* or *at a point in space* the question arises: Which kind of function is being used here? The only answer consistent with the truth conditions of (24) seems to be that the temporal function is being used: The width measurement in (24) is true at some contextually available past instant of time over an entire spatial interval.[12] The spatial interval is being determined by an axis of reference running from the north end to the trail head, as described by a spatial path phrase in our sense. Thus we have width as a function of time co-occurring with a spatial path phrase.

We make the following assumptions about the adjective *wide*:

(a) It is lexically specified to take spatial paths (temporal paths are out, because there is no motion).
(b) The width function may be either temporally or spatially indexed.
(c) Let **width** be a primitive width measurement function giving widths of spatial regions. Then we assume:

(a) $\text{wide}_{\text{S}}(\sigma)(s) = \text{width}(\text{path}_{\text{S}}(\sigma)(s))$

(b) $\text{wide}^{\text{S}}_{\text{T}}(\sigma)(t) = \text{width}(\text{path}^{\text{S}}_{\text{T}}(\sigma)(t))$

This path operator in (b) is neither the $\textbf{path}_{\mathrm{S}}$ in (a) nor the $\textbf{path}_{\mathrm{T}}$ introduced above. It is a way of defining a temporally indexed path function for a predicate which is basically a spatial path predicate. The definition is:

$$\textbf{path}^{\mathrm{S}}_{\mathrm{T}}(e)(t) \quad = \quad \mathrm{AT}(\mathrm{theme}(e), t) \sqcap \mathrm{Loc}(\mathrm{path}_{\mathrm{S}}(e))$$

Loc is a function returning the entire spatial region covered by a path function, defined as:

$$\mathrm{Loc}(\pi) = \bigsqcup_{s \in \mathrm{Dom}(\pi)} \pi(s)$$

For any time t, $\textbf{path}^{\mathrm{S}}_{\mathrm{T}}(e)(t)$ is the location of the theme of e restricted to the interval determined by the spatial path of e. This, then, is a time sensitive path function that does not entail motion. If the theme of e is a wall and the path is restricted to be from the north gate to the tower, $\textbf{path}^{\mathrm{S}}_{\mathrm{T}}(e)(t)$ returns the portion of the wall between the north gate and tower at time t.

We thus assume that the semantics of (24) is:

(25) $\exists\sigma[\mathrm{wide}^{\mathrm{S}}_{\mathrm{T}}(\sigma)(t)=[6\,\mathrm{ft}] \wedge \mathrm{theme}(\sigma)=c \wedge [n:t] \circ \textbf{path}_{\mathrm{S}}(\sigma)]$

Here t is a contextually provided time index. The path operator in (25) will require that the path of state σ run along some spatial axis that places σ's start in some location overlapping the North end and σ's end in some location overlapping the trail head at time t.

This completes the account of the semantics of the adjective *wide* with paths. We extend the account to *widen* simply by assuming that the $\mathrm{INCREASE}_{\mathrm{T}}$ operator preserves the spatial path of the start and end states. This is guaranteed by the following modified version of axiom (14):

(26) $$\forall e, d \left[\begin{array}{l} \mathrm{increase}_{\mathrm{I}}(\mathrm{wide}^{\mathrm{S}}_{\mathrm{I}})(e) = d \\ \qquad\quad \longleftrightarrow \\ \exists\sigma_1, \sigma_2[\ \mathrm{START}_{\mathrm{I}}(\sigma_1) = \mathrm{START}_{\mathrm{I}}(e) \wedge \mathrm{END}_{\mathrm{I}}(\sigma_2) = \mathrm{END}_{\mathrm{I}}(e) \wedge \\ \qquad\quad \mathrm{wide}^{\mathrm{S}}_{\mathrm{I}}(\sigma_1)(\mathrm{END}_{\mathrm{I}}(e)) = \mathrm{wide}^{\mathrm{S}}_{\mathrm{I}}(\sigma_2)(\mathrm{START}_{\mathrm{I}}(e)) + d \wedge \\ \qquad\quad \textbf{path}_{\mathrm{S}}(\sigma_1) \subseteq \textbf{path}_{\mathrm{S}}(e) \wedge \textbf{path}_{\mathrm{S}}(\sigma_2) \subseteq \textbf{path}_{\mathrm{S}}(e)] \end{array} \right.$$

This axiom basically states that to extend width states out along any axis their axes of reference S must be the same as that used by e and their spatial paths must be extended. The change to axiom (14) is that the requirement that the figures of

σ_1, σ_2 and e be the same has been strengthened to the requirement that the spatial paths be subsets of σ_1, σ_2 be subsets of the spatial path of e.

Thus the definitions of *wide* and *increase* lead to an immediate account of the semantics of path expressions with *widen*, illustrated in (27b), which gives the extent reading for (27a):

(27) a. The crack widened 5 inches from the North gate to the tower.

b. $[\![\exists\, e\ \ [\text{increase}_\text{S}(\text{wide}_\text{S})(e)\text{=}[.5\,\text{in}]\ \wedge\ \text{figure}(e)\text{=}c\ \wedge$
$[\text{ng : t}] \circ \textbf{path}_\text{S}(e)]]\!]$

The path expressions constrain the path which in turn determines the domain over which the change measurements are taken. The minimal point of e along axis S must overlap the north gate and the maximal point must overlap the tower. The difference in width between those two extremes must be a half-inch.

Summing up, in this section I have proposed an analysis of spatial and temporal paths that accounts for both verbs of motion and extent verbs. The analysis extends naturally to account for the use of path phrases with stative predicates like the adjective *wide* and degree achievements like the verb *widen*.

Note that the domain and range of the path function of *widen* has not changed in this section. It is still a mapping from indices to widths. What has changed is that the measurements are now being constrained by the path-function. Thus we have simply recast *wide* as a function that measures widths along a path.

Gawron (2006) argues that the analysis of paths explains why extent readings with degree achievements are restricted to predicates with spatial paths. Summarizing, INCREASE must apply to a predicate which is stative relative to the axis of change. On an extent reading that axis is some spatial axis S, and this turns into the requirement that the predicate be cumulative relative to the path axis. A version of cumulativity called path-cumulativity is proposed, revising the definition of cumulativity in Zwarts (2005). In order for a predicate to be path cumulative it must be defined relative to a spatial axis; and the only descriptive device language supplies for specifying spatial axes is spatial path expressions. Thus to be eligible for INCREASE_S a predicate must be path cumulative; to be path-cumulative a predicate must in turn be compatible with spatial path expressions.

3 Aspectual variation

In this section we discuss the two cases of aspectual variation described in the introduction, variation in graduality and variation in incrementality.

3.1 Graduality

We return to the varying behavior of *gradually* in the following examples, reproduced from Section 1.

(28) a. The crack gradually widened from the North gate to the tower.
b. Fog gradually covered the peninsula from the pier to the point .

The issue is that (28a) has both an event and an extent reading; but (28b) has only an event reading. The question, then, is: Why aren't extent readings for *cover* compatible with *gradually*?

There is one account that is not an option. We cannot say that that *gradually* does not combine with *cover* because *cover* has an end of scale degree predicate in it. The fact is that *gradually* does combine with *cover* on the event reading, and if extent *cover* is an end-of-scale predicate, then surely event *cover* is as well.. The verb *cover* is just like other incremental theme predicates: In each sub event the part of the theme that is covered is completely covered, just as the part of the apple that is eaten is eaten; matters progress because parts that are completely covered can belong to larger things that are not, and *gradually* is quite compatible with this kind of progress. The question is: Why does that kind of progress count as progress along the temporal axis but not on the spatial axis?

Another kind of account that is not an option is to say that *gradually* is incompatible with the notion of change along a spatial axis; *gradually* works quite well with the extent reading of *widen* in (28a).

What then distinguishes the extent reading *cover* from the extent reading of *widen*? First of all, notice that in contrast to *widen* and in contrast to *cover* on event readings, extent *cover* shows no evidence of being an accomplishment:

(29) a. # The snow covered 100 square miles of canyon in just 5 miles.
b. The snow covered the canyon in 5 minutes.
c. The crack widened an in inch in 5 yards.

I would like to suggest that the key difference is that *cover* on the extent reading, despite being a verb, expresses a state: The problem is that *gradually* requires a verb of gradual change, and while extent-*widen* falls into that class, extent-*cover* does not. Within the parameters of the current account, the most natural way to capture this is to say that the verb *cover* does not combine with the $\textsc{increase}_S(\text{cover}_S)$ operator, and that what appears to be the extent reading is just a stative use. The resulting differences between *widen* and *cover* are shown in (30):[13]

(30)

	Reading	Form	Semantics
(a)	Stative	[$_V$ cover]	$\text{cover}^{\text{S}}_{\text{T}}(e)(t),\ \text{cover}_{\text{S}}(e)(s)$
(b)	Event	[$_V$ cover]	$\textsc{increase}_{\text{T}}(\text{cover}^{\text{S}}_{\text{T}})(e)$
(c)	Extent	*	$\textsc{increase}_{\text{S}}(\text{cover}_{\text{S}})(e)$
(d)	Stative	[$_A$ wide]	$\text{wide}^{\text{S}}_{\text{T}}(e)(t),\ \text{wide}_{\text{S}}(e)(s)$
(e)	Event	[$_V$ widen]	$\textsc{increase}_{\text{T}}(\text{wide}^{\text{S}}_{\text{T}})(e)$
(f)	Extent	[$_V$ widen]	$\textsc{increase}_{\text{S}}(\text{wide}_{\text{S}})(e)$

First, we motivate the idea that the extent reading of *cover* is due to stative semantics paralleling an adjective like *wide*. Is there independent evidence for a transitive stative *verb* with scalar semantics? For one, we have verbs like *love*:

(31) a. John loves/? is loving Mary.
b. John loves Mary a lot/a bit/slightly.
c. John loves Mary more than Sue loves Tom.

The incompatibility of *love* with the progressive illustrated in (31a) establishes its credentials as a stative verb; and (b) and (c) illustrate it in use with degree-sensitive modifiers. Note in particular that (c) has an intensity reading not available for other verbs in superficially similar comparatives. These generally have frequency readings only:

(32) a. John hit Mary more than Sue hit Tom.
b. John hit Mary harder than Sue hit Tom.

(32b) but not (32a) has an intensity reading in virtue of introducing an overt degreeable predicate. (31c) requires no such addition for an intensity reading.

The key claim of (30) with respect to graduality is that *cover* only combines with INCREASE when it uses a temporal axis of change, so that, with the degree achievement semantics, only the event reading is possible. This is essentially the analysis of Jackendoff (1990), with the INCREASE operator replacing his BECOME operator. The chief objection to that analysis, registered in Gawron (2006), was that it did not capture the fact that the aspectual natures of the predicates with event and extent readings were basically the same. But here, the data in (29) and the co-occurrence facts with *gradually* clearly argue that event and extent readings of *cover* DO have different aspectual natures, so that objection goes away.

But though this makes the account work, we are left with essentially the same question we started with. Why? What explains the gap in (30c)? Why should $\text{INCREASE}_{\text{T}}$ combine with COVER when $\text{INCREASE}_{\text{S}}$ will not.

To explain this, let us consider a specific account of *cover* consistent with the approach taken here.

Dowty (1991) points out that cover-verbs are incremental theme verbs. In fact, cover verbs have two participants that can qualify as incremental themes. Let us call the the snow and the mountain in (35b) the theme and goal respectively. As a cover-event progresses, more and more of the goal's surface is covered; but so also is more and more of the theme's surface moved over the goal. Progress in the event requires simultaneous consumption of two areas.

A natural lexical predicate capturing this basic semantic fact is the following:

(33) $\text{cover}_{\text{S}}(e) = \pi$ iff $\text{path}_{\text{S}}(e) = \pi$ and $\text{cover-path}_{\text{S}}(e, \pi)$
where $\text{cover-path}_{\text{S}}(e, \pi)$ iff $\text{Loc}(\text{ON}_{\text{S}}(\text{goal}(e))(\mathcal{T}(e))) \sqsubseteq \text{Loc}(\pi)$

ON_{S} is one of a family of path functions incorporating different spatial relations, as path_{S} incorporates AT.[14] I assume the underlying spatial function ON returns the spatial region on or above its argument at a time t. A covering event cannot have just any spatial path; it must have a path on which each slice of the theme covers the corresponding slice of the goal.

We assume *cover* is a basic spatial path predicate just as *wide* is. To derive a temporally indexed version, we use $\text{path}_{\text{T}}^{\text{S}}$ just as with *wide*. Defining a predicate $\text{cover}_{\text{T}}^{\text{S}}$ subject to the same covering path constraint but using $\text{path}_{\text{T}}^{\text{S}}$ gives us:

(34) $\text{cover}_{\text{T}}^{\text{S}}(e) = \pi$ iff $\text{path}_{\text{T}}^{\text{S}}(e) = \pi$ and $\forall t \in \mathcal{T}(e)[\mathcal{G} \sqsubseteq \text{path}_{\text{T}}^{\text{S}}(e)(t)]$
where $\mathcal{G} = \text{Loc}(\text{ON}_{\text{S}}(\text{goal}(e))(\mathcal{T}(e)))$

This semantics for $\text{cover}_{\text{T}}^{\text{S}}$ parallels the temporally indexed semantics of *wide*, $\text{wide}_{\text{T}}^{\text{S}}$; that is, both incorporate $\text{path}_{\text{T}}^{\text{S}}$, and thus both are stative predicates evaluable at instants of time. The INCREASE operator will be required to introduce change in the amount of covered area in time. The semantics for an event reading of *cover* is:

$$\text{INCREASE}_{\text{T}}(\text{cover}_{\text{T}}^{\text{S}})$$

The analysis of the basic meaning of *cover* parallels the analysis of the basic meaning of *extend*; it is a stative predicate whose state function is composed from **path**; it is therefore another example of change by parts. Since $\text{path}_{\text{T}}^{\text{S}}$ returns the location of theme at each moment t, restricted by the spatial path, and since INCREASE requires that area to be increasing with respect to the sub-region relation, this captures the fact that eventive *cover* describes spreading motion of the theme in examples like (1d).

The crucial point for our development is that $\text{cover}_{\mathrm{S}}$ and $\text{cover}^{\mathrm{S}}_{\mathrm{T}}$ return very different kinds of things when applied to their appropriate indices. For a given time t, $\text{cover}^{\mathrm{S}}_{\mathrm{T}}(e)(t)$ returns the entire portion of the theme's location that is on the goal at t; while for a given spatial index s, $\text{cover}_{\mathrm{S}}(e)(s)$ returns the slice of the theme at s that is on the goal.

As desired, (33) implicates both the theme and the goal in the state function. Clearly, the $\text{ON}(\mathrm{Goal}(e))$ term is motivated because covers overlap the surface of their goals. The theme's location might have been introduced any number of ways: The basic motivation for introducing it by way of *path* is that, like the other event/extent verbs *cover* also takes path expressions:

(35) Snow covered the mountain from the valley floor to the summit.

(33) explicitly states the semantics of covering incorporates the semantics of pathhood.

Now let us return to the issue of graduality. I have proposed that the reason extent readings for *cover* do not show graduality is that they do not incorporate an increase operator. That is, the semantics of the verb *cover* on an extent reading is like the semantics of an adjective like *wide*. On the other hand, the semantics of *cover* on an event reading DOES include an INCREASE operator. What makes a stative predicate like *wide*, which can combine with both $\text{INCREASE}_{\mathrm{T}}$ and $\text{INCREASE}_{\mathrm{S}}$, different from the stative predicate *cover*, which combines only with $\text{INCREASE}_{\mathrm{T}}$?

The answer lies in the definition of the increase operator. Schematically, what is required for a state predicate α is the following:

$$\begin{aligned}\text{increase}(\alpha)(e) = d \;\; \text{iff } \exists \sigma_1\, \sigma_2\;\; & \alpha(\sigma_1) = d_1 \;\wedge\; \text{START}(\sigma_1) = \text{START}(e) \wedge \\ & \alpha(\sigma_2) = d_2 \;\wedge\; \text{END}(\sigma_2) = \text{END}(e) \wedge \\ & d_1 + d = d_2\end{aligned}$$

In order for d to be uniquely defined with d_1 and d_2 fixed, the "+" operation here needs to be the inverse of some difference operation. That is, there needs to be a uniquely defined operation "-" such that:

$$d_2 - d_1 = d$$

In sum, the range of the state function α must be some domain for which a **difference operation** can be defined.

At least two properties are required. First, there clearly must be some kind of partial ordering. Second, we can identify what has been called the **Remainder Principle** (Krifka 1998).

(36) **Remainder principle:**

$$\forall x, y \in \mathrm{U_P}[x <_{\mathrm{P}} y \rightarrow \exists! r[\neg[r \otimes x] \wedge x \oplus r = y]]$$

Here $\mathrm{U_P}$ is the set of elements in an ordered structure P and $<_{\mathrm{P}}$ is the proper partial order on those elements, excluding equality, $\otimes$ is the overlap relation (x and y overlap if they share a common lower bound), and $\oplus$ is the join (summing) operation. The Remainder Principle requires that for any two ordered elements, $x < y$, there exists a remainder r which does not overlap x and which can be summed with x to give y. In a word, r is the difference between x and y.

The Remainder Principle is satisfied by degrees on a scale. If $d_1 < d_2$, there is some minimal d such that

$$d_1 + d = d_2$$

This d does not over lap d_1

But (36) may also be satisfied by what is called **a mereology**, a part-whole structure. The simplest conception of a mereology is as a join-semi-lattice with an overlap relation. A further natural requirement that may be placed on a mereology is the Remainder Principle. For example, Link's 1983 algebra of mass terms and plurals are both mereologies satisfying the remainder principle; so are locations under the sub-region relation. This is why we were able to make sense of applying INCREASE$_{\mathrm{T}}$ to path$_{\mathrm{T}}$ in Section 2.2.2. Axioms for a mereology including the above remainder principle are presented in Krifka (1998:199), as adapted from Simons (1987), and are reproduced in the appendix. What is relevant for our present concerns is that while both cover$_{\mathrm{S}}$ and cover$^{\mathrm{S}}_{\mathrm{T}}(e)(t)$ take their ranges in the mereology of locations, cover$_{\mathrm{S}}$ does so only trivially. No two elements in the range of cover$_{\mathrm{S}}$ are ordered because the range of cover$_{\mathrm{S}}$ is a set of disjoint slices. Thus it is quite natural that INCREASE cannot apply to cover$_{\mathrm{S}}$. On the other hand, the range of cover$^{\mathrm{S}}_{\mathrm{T}}(e)$ in a world of continuous motion must include spatially overlapping regions. Thus it is quite natural that INCREASE can apply to cover$^{\mathrm{S}}_{\mathrm{T}}(e)(t)$. We flesh these ideas out in Section 4 by defining the class of verbs of gradual change, using as one of the conditions the remainder axiom.

Contrast the state functions for *wide*. Whether temporally or spatially indexed, the function *wide* takes as its range a set of degrees which obey the Remainder Principle. Thus both INCREASE_S and INCREASE_T may apply to it, producing spatial and temporal accomplishments.

The case of *zigzag* is quite similar. Although there is no INCREASE operator in its definition it is defined through a degreeable state function which takes its range in a set of degrees. Once again the remainder principle will be satisfied and *zigzag* will qualify as a verb of gradual change and be eligible for modification by *gradually*.

With respect to graduality, the account of *cover* is virtually identical to our account of *extend* in (18) . Like *cover*, *extend* is compatible with *gradually* only on the event reading. Like *cover*, *extend* uses a temporally indexed path which returns locations and therefore describes change by parts. Thus, like $\text{cover}^\text{S}_\text{T}$, extend_T incorporates with INCREASE_T to give event readings describing spreading motion. extend_T is eligible for combination with INCREASE_T and therefore has event readings describing spreading motion.

Summing up the results of this section: We have accounted the property of graduality in terms of the property of having a state function with a mereological range. This requirement, which we have argued is a natural requirement for characterizing gradual change, accounts for the fact that verbs like *cover* and *extend* exhibit graduality only on event readings. In contrast verbs that take their range in degrees, like *zigzag* and *widen*, can be gradual on both event and extent readings.

3.2 Incrementality

In this section, I look at whether path is really an incremental theme (Dowty 1991) in event readings; that is, do the truth conditions require that the path covered grow homomorphically with the event, with the location identified in the *from* phrase overlapped at the beginning of the event, and the location identified in the *to*-phrase overlapped at the end?

(37) Incrementality

[+ Incr_e]	(a)	A storm front zigzagged from Prescott to the border.
	(b)	The fog extended from the pier to the point.
[- Incr_e]	(c)	The crack widened from the tower to the north gate.
	(d)	Fog covered the peninsula from the pier to the point

For the cases marked [- Incr_e], the answer is no. In particular, on the non-incremental event reading of (c) the progression of the crack's widening may be in any order, say, from gate to tower, as long as the event concludes with a widening that covers that span; and in (d) the fog's progress may be in any order as long as in the end a span between pier and point is covered.

The account presented in the previous sections of the semantics of the four predicates in (37) predicts these facts. The key point is that temporal paths are temporally incremental and spatial paths are not. Spatial paths serve to identify a spatial interval for an event, not to track the location of the theme through the event's course. Thus the relevant property distinguishing the [+ Incr_e] predicates from [- Incr_e] predicates is that [+ Incr_e] predicates allow spatial paths on their event readings.

The compatibility of spatial paths with predicates that incorporate the INCREASE_T operator is guaranteed by axioms like (26), which was used for *widen*. It relies on the spatial-path being well-defined for the stative predicate INCREASE_T applies to, and for predicates like *wide* and *cover* this is well motivated.

Equally important is to guarantee the *incompatibility* of spatial paths with [+ Incr_e] predicates on their event readings. On the current analysis, this follows for *extend* for the uninteresting reason that the increase operator applies to path_T to yield the verb's event readings, so no predicate allowing spatial paths is defined. The case of *zigzag* is more interesting. The predicate *zigzag* is defined as predicate with a single axis of change which may be either spatial or temporal but not both. Neither the event nor extent reading requires appeal to an INCREASE operator, and the *increase* operator is not defined for functions of events, so that it cannot apply to either version of *zigzag*. Thus, there is no constructive device for introducing a second axis into the semantics. Choosing an event or extent reading necessarily entails choosing a temporal or spatial path. In the case of an event reading, this necessarily entails an incremental path.

Note that incrementality is a concept that makes sense for both spatial and temporal axes of change. This account makes the prediction that INCREASE_S applied to path_S predicates will also yield predicates that are incremental on the extent reading, since the path phrase domain and the axis of change are the same. Thus, in (37c), for example, on the extent reading, the path-phrases do impose a directionality on the change; the crack must get wider as we move in the direction from the tower to the gate, not the other way. On extent readings, the path phrase of *widen* IS incremental, yet on event readings it is not. The facts and argument for *zigzag* are parallel.

4 Dimensionality and change in lexical structure

The analysis given in this paper has basically centered on four predicates *widen extend*, *zigzag*, and *cover*, which between them partition the range of variation of predicates with event/extent ambiguities. Our central project had been to account for event/extent ambiguities and, to that end, we have used two distinct mechanisms. There are verbs like *zigzag* and *widen* which simply measure change of a certain kind and can use either a temporal or spatial axis to measure it on. These verbs fall in the same aspectual class for both event and extent readings. There are also verbs which require an INCREASE operator for the event readings and are spatial states (0-dimensional predicates) on their extent readings. These are verbs like *cover* and *extend*. This is shown in (38):

(38)

Verb	Extent	Event	
widen	$\text{INCREASE}_{\text{S}}(\text{wide}_{\text{S}})$	$\text{INCREASE}_{\text{T}}(\text{wide}^{\text{S}}_{\text{T}})$	Uniform Aspect
zigzag	$\text{ZIGZAGGY} \circ \text{path}_{\text{S}}$	$\text{ZIGZAGGY} \circ \text{path}_{\text{T}}$	
extend	path_{S}	$\text{INCREASE}_{\text{T}}(\text{path}_{\text{T}})$	Aspect Change
cover	$\text{cover}^{\text{S}}_{\text{T}}$	$\text{INCREASE}_{\text{T}}(\text{cover}^{\text{S}}_{\text{T}})$	

We have thus assumed that there is aspectual variation to account for. In this section we summarize the analysis, develop a general characterization of gradual change, and relate it to the idea of dimensionality, largely as developed in Jackendoff (1996).

We accounted for two aspects of aspectual variation, incrementality and graduality.

Since our discussion of incrementality basically concerned its effects on event readings, we can summarize our account by collecting together our event readings. This is done in (39):

(39)

Verb	Δ_e	Path	
zigzag	$\text{zigzaggy} \circ \text{path}_{\text{T}}$	path_{T}	[+ Incr$_e$]
extend	$\text{INCREASE}_{\text{T}}(\text{path}_{\text{T}})$	path_{T}	
widen	$\text{INCREASE}_{\text{T}}(\text{wide}^{\text{S}}_{\text{T}})$	$\text{path}^{\text{S}}_{\text{T}}$	[- Incr$_e$]
cover	$\text{INCREASE}_{\text{T}}(\text{cover}^{\text{S}}_{\text{T}})$	$\text{path}^{\text{S}}_{\text{T}}$	

The account is that event readings with temporal paths will require incrementality of the path. All extent predicates allow spatial paths, but some also allow temporal paths. Since temporal paths can only occur on event readings, those predicates will have incremental paths on event readings.

The issue with graduality was the compatibility of *gradually* with certain predicates on their extent readings. We can summarize our account by collecting our extent readings. This is done in (40):

(40)

Verb	Δ_e	Δ_t	
widen	$\text{INCREASE}_{\text{S}}(\text{wide}_{\text{S}})$	NA	[+ Grad $_e$] (degrees)
zigzag	zigzaggy $\circ$ path_{S}	NA	
extend	NA	$\text{path}^{\text{S}}_{\text{T}}$	[- Grad $_e$] (parts)
cover	NA	$\text{cover}^{\text{S}}_{\text{T}}$	

Our account is that predicates describing change by parts were not compatible with the $\text{INCREASE}_{\text{S}}$ operator and could only be spatially stative on their extent readings. This in turn was due to the fact that spatial paths return slices, which are not ordered with respect to each other, and do not provide state functions (Δ_ts) with the right sort of range for INCREASE to operate on.

We now show how the graduality facts can follow from a general characterization of verbs of gradual change, building on the ideas about mereologies of Section 3.1

(41) **Verbs of gradual change**

If α is the predicate of a verb of gradual change then there exists some mereology M such that:

$$\forall e,\ f[\ \alpha(e){=}f\ \rightarrow \text{dynamic}_{\text{M}}(f)]$$

where a function is $\text{dynamic}_{\text{M}}$ if and only if it

(a)

$$\text{Range}(f) \subseteq \text{M};$$

(b) Change in f nontrivial:

Non-trivial change:

$$\forall x \in \text{Range}(f)\ \exists y \in \text{Range}(f)[\ \ x <_{\text{M}} y \vee y <_{\text{M}} x\ ]$$

The axioms for a mereology are in the appendix, but the key requirements are:

(1) M is a join semi-lattice;
(2) M obeys the remainder principle of (36)

Examples of mereologies include sets under the subset ordering, masses of stuff under the consists-of ordering, paths under the subpath ordering, and locations under the subregion ordering. Note also that mereologies include sets of degrees as a special case.

The account of graduality, then, is simply that the adverb *gradually* has as a necessary condition that the predicates it combines with be predicates of gradual change. The two non gradual verbs *cover* and *extend* both have basic state meanings. In order to acquire Δ_e functions they must combine with INCREASE. But they also both describe change by parts because their state functions return locations. When they are built on path_{T} or $\text{path}_{\text{T}}^{\text{S}}$, an event with spreading motion will have a state function satisfying the non-triviality requirement. When they are built on path_{S}, however, the range of the state function is a set of disjoint spatial slices and non-triviality cannot be satisfied. Therefore there can be no extent reading with the semantics of a spatial accomplishem,ent or activity, and *gradually* is not a possible midifer.

The INCREASE operator does not apply to either version of *zigzag*. Why, for example, can't $\text{INCREASE}_{\text{T}}$ apply to $\textit{zigzag}_{\text{T}}$ to produce a second-order version of *zigzag*, one which describes an event in which the degree of zigzaggyness changed?

The basic intuition is that it can't apply because a verb can have only one axis of change, and *zigzag* already has its one allowed axis of change. The formal reflex of this intuition is that $\text{INCREASE}_{\text{S}}$ must apply to functions from events to spatially indexed state functions and $\text{INCREASE}_{\text{T}}$ must apply to functions from events to temporally indexed state functions. That is, both apply to denotations approrpiate for states, 0-dimension predicates. The verb *zigzag*, on the other hand, denotes a function from events to degrees, a function appropriate for an accomplishment or activity, which returns a degree measuring the entire course of change in an event.

Summing up, non-triviality imposes a requirement defining gradual change, and that definition presupposes an axis of change. A verb can have only one axis of change. All the examples above where two axes are used are cases where one axis was the axis of reference and the second an axis of change. When events with change in two spatial dimensions have arisen (*cover*), I have described them with state functions whose values are surfaces. Thus the structure of the account is that there is always one independent variable, the axis of change, which can be mapped to a variety of domains some of them with multiple spatial dimensions.[15]

5 Verbs of gradual change: The bigger picture

In looking at a narrowly defined class of predicates exhibiting event/extent ambiguities, we have reached some rather general conclusions about change. In particular, we have proposed the following condition for verbs of gradual change:

(42) **Verbs of gradual change**

If α is the predicate of a verb of gradual change then:

$$\forall e,\ f[\,\alpha(e)=f\ \rightarrow \text{dynamic}_{\text{M}}(f)]$$

where dynamic$_{\text{M}}$ is true of a function f if and only if its range is a mereology and it exhibits nontrivial change.

In this section we try to relate this proposal to the general problem of characterizing telicity for verbs of gradual change, in particular, to the Degree Hypothesis articulated in Kennedy and Levin (2001).

We can very broadly identify two approaches to gradual change, mereological and degree-based, approaches which may in the end be not very incompatible but which at the very least have emphasized different sets of data.

A useful place to begin a discussion of mereological approaches is with the concept of *incremental theme*. Dowty (1991) introduced the term incremental theme to describe a participant incrementally affected as an event progresses. The class of participants who "delimit or measure out the event" (Tenny 1994) is closely related. It is worth reviewing some of the basic ideas to see how the entry of degrees into the picture is motivated.

Some examples of incremental theme verbs are given in (43), with the incremental themes italicized:

(43) a. John ate *the bagel* (in 5 minutes).
b. Mary learned *the sonata* (in 5 days).
c. Beethoven wrote *a sonata* (in 5 days).
d. Alice mowed *the lawn* (in 5 minutes).
e. Bobbie Joe read *War and Peace* (in 5 days).

The unifying property of these verbs has been succinctly described by Krifka (1989):

> **Incremental Theme Verbs**
> If an incremental theme verb expresses a relation R and R holds in event e and $\Theta(e)$ is the incremental theme, then in any sub-event of e in which R holds a portion or part of $\Theta(e)$ stands in the relation R.

For example, in any sub-event e' of a bagel eating e some portion b' of the bagel b has been eaten and in any eating sub-event e'' of e' some portion b'' of b' has been eaten, and so on. Correspondingly all the clauses become atelic when that participant is given an unquantized (Krifka 1989) or unbounded description:

(44) a. John ate bagels in 5 minutes (habitual reading only)
b. Mary learned sonatas in 5 days (habitual only)
c. Beethoven wrote sonatas in 5 days (habitual only).
d. Alice mowed lawns in 5 minutes (habitual only)
e. Bobbie Joe read novels in 5 days (habitual only)

Krifka's 1998 observation is that the theme role is a homomorphism from events to individuals preserving the part-of relation. Thus in the case of the verb *eat*, for example, the incremental theme bounds the event in the following sense: When the incremental theme is completely affected, the event is complete. How general is feature among telic verbs? Clearly not very, as both Krifka and Dowty note. Krifka cites examples such as an event of house-building which may involve various preparatory activities such as digging a large hole for a foundation, a subevent with subparts not directly related to any part of the finished house. He also cites the case of *peeling an apple* in which not the entire theme but only a portion of it is incrementally affected.

Other authors have pointed out further cases in which part/whole relations play no part. Rothstein (2004) cites the following:

(45) a. repair the computer
b. prove the theorem
c. solve the Rubik's cube

The relevance of degree acheivements to the discussion is that they offer a rich stock of cases in which telicity is determined with no reference at all to parts. For example, a suitcase closing (Filip 1999) does not progress by having more and more of the suitcase closed, but by having the suitcase more and more closed; soup is generally cooled by lowering its average overall temperature, not part by part (Levin 2000). Interestingly, such verbs also exhibit an accomplishment activity ambiguity:

(46) a. The soup cooled for 3 minutes (but was still too hot).
b. The soup cooled in 3 minutes

This is the point of departure for the Hay et al. (1999) analysis. They account for such ambiguities with a semantics that contains a degreeable predicate as a component, and attribute telicity to pragmatically supplied bound on the degree.[16]

Degree achievements provide a large class of cases in which there is both syntactic and morphological evidence for the existence of a scale. How much evidence is there for the existence of a scale outside of that class of verbs?

The answer is that there is quite a bit. Zucchi (1998) argues convincingly that the aspectual ambiguities of a verb like *bake*, similar to those exhibited by degree acheivement verbs, may be accounted for assuming a degreeable predicate in the semantics. Kennedy and McNally (1999) point to another class of cases which also provides morphological and syntactic evidence, derivations from verb to adjective. If a participial adjective is derived from an incremental theme verb, it has a closed scale, as in *a partially eaten muffin*. Similarly, Kratzer (2000) assumes 'target states' for verbs in her account of adjectival passive constructions in German and English, and assumes that the scalar properties of those derived adjectives are predicted by the scalarity of the target states of the verb. In both the Kennedy-McNally and Kratzer analyses, there needs to be a way to get from telic events to closed adjectival scales and the assumption that the verb includes a degreeable predicate provides that way.

One issue is: What is the scope of the account? Are all predicates internally degreeable? Hay et al. (1999) explicitly argue for extending a degree account to the class of incremental theme verbs, suggesting for example, that for a verb like *eat*, telicity is directly linked to the size or volume of the theme. Indeed, it seems to be possible to extend the degree account to handle incremental theme verbs, but not to extend the mereological account to handle degree achievemnt verbs. Kennedy and Levin (2001) propose to restrict the degree account to what they call verbs of gradual change, the term that has been adopted throughout here, but they clearly intend to include incremental theme verbs like *eat*. They examine, among other pieces of evidence, the use of measure adverbials like *a bit* and maximizing adverbials like *completely, totally* and *halfway* with telic verbs. Beavers 2004, Wechsler 2005 extend the use of degrees in trying to account for the distribution of adjective phrases and path phrases in resultatives, arguing that the scales associated with resultatives must be the kind of scales compatible with telicity (closed). In doing so they assume that resultatives always have associated scales, and that path phrases do as well.

Thus we have a body of work pointing to the conclusion that for at least a large class of verbs, there is a semantic component that is a degree which corresponds to the degree of change in the described event. Henceforth in this section the object of attention is what I will call the Degree Hypothesis. The following quote is from Kennedy and Levin (2001):

> **The Degree Hypothesis** Verbs of gradual change contain gradable properties as part of their meaning. Telicity is determined by the semantic properties of the degree of change.

I interpret the term "verbs of gradual change" to mean almost all accomplishments and all activities, excluding a class of verbs describing abrupt transitions like those in (45).

In order to get more specific about the degree hypothesis, I shall assume that a class of activities and accomplishments simply are quantizable properties of events (sidestepping for the moment such issues as what to do about agents). That is I assume every degreeable verb α has a **change function** Δ_e for which it is sensible to write:

(47) $\Delta_e(e) = q$

I will refer to q as the *change argument*.[17] The claim HKL are making then, assuming they admit Davidsonian semantics and would assent to something like (47), is that the change argument must be a degree.

In the discussion in this paper I have assumed a somewhat larger class of kind of change functions, allowing functions with either degree sets or parts as their ranges. The resulting mathematical structures — mereologies — can be thought of as generalizations of simple degree systems. In particular, since the mereologies assume here obey the remainder axiom, something like the INCREASE operator can be defined.

Bringing mereological state functions into the fold allows the following analyses.

1. Verbs of motion. Instead of having change argument be a degree on a scale (the odometer distance traveled), it is the path itself, assuming a model of paths (like the one assumed here) in which there is a partial order in paths. In the degree case the change argument is filled by distance phrases like *5 miles*; in the other, by path phrases like *from Boston to Paris*. Either kind of modifier, if bounded, makes the event description bounded and telic, and either bounds the other, so the choice is not obvious.

2. Verbs of consumption. Instead of the change argument being measure of volume, it is chunks of stuff. Again, bounding either the volume of the theme or the theme bounds the event. In terms of defining the state function, it is either

$$\lambda e.\lambda t.\text{consists-of}(\text{theme}(e),\ t)$$

or

$$\lambda e.\lambda t.\text{volume-of}(\text{consists-of}(\text{theme}(e),\ t))$$

3. Cover/fill verbs. Instead of the change argument being a degree of coveredness, it is the spatial region actually covered, the intersection of regions associated both with the theme and the goal.

Note that in all three cases, the non-scalar options introduces a domain in which part-of relations apply, paths, locations, and mass.

Now there is a critical difference between scales and mereologies. Mereologies, being *partial* orders, cannot offer an account of comparison. Consider:

(48) a. This tub is more full than that one.
b. This tub filled more than that one.

What ordering licenses the comparisons in (48)? The *part-of* relation imposes no order at all on the masses of water in the two tubs, since neither is part of the other. Nor is the ordering relevant to the comparison given by their volume measures, since a small tub may be fuller than a large one containing twice the volume of water. Thus, the fullness ordering is given by a measure something like *percentage of goal's volume filled*, clearly a linear ordering. This is an immediate problem for the defender of a mereological account of the verb *fill*, especially (b), since it appears to be a direct comparison and it appears to be happening with the verb.[18]

Example (48b) is relevant to the analysis here on the assumption that its semantics really involves a direct comparison of the change arguments of the two instances of *fill*. The existence of *verbal comparatives* of this sort is clearly motivated for uncontroversial degreeable properties like the following:

(49) a. Highway 8 widens more than Highway 5 in that stretch.
b. John loves Mary more than Fred does.

Notice there is a nonfrequentative reading which just compares the relevant degrees of the two predicates, and that *more* does not usurp the position of any NP arguments. I take it the existence of an example like (49b), for example, is a major obstacle for a hypothetical mereological account of *love*, in which the state functions for (49b) would return quantities of John-love and Fred-love not ordered by a part-of relation, as I consider (48b) is a major obstacle for a mereological analysis of *fill*.

Now compare examples with *cover* and *fill*:

(50) a. By late afternoon the shadows covered the patio more than the garden.
b. By late afternoon the shadows covered more of the patio than of the garden.
c. The tub in the guest bathroom filled more than the tub in the master bathroom.
d. ? More of the tub in the guest bathroom filled than (of) the tub in the master bathroom.

Examples (a) and (b) are both acceptable, though (a) may be slightly awkward. Somewhat surprisingly, matters are completely skewed for *fill*; the verbal comparative is clearly the more natural construction. I take it this is strong evidence that the verb *fill* uses a linearly-ordered fullness-scale, and that (d) is unnatural for the same reason (51a) is a very marked way of saying (51b); (51a) is natural only when there is some salient reason why half the tub is a separate entity, such as a wall dividing the tub in two.

(51) a. Half the tub was full.
b. The tub was half full.

What this suggests is that the *theme* of *fill* is not at all an incremental theme in the sense of (Krifka 1998). If x is an incremental theme for an event e, then sub-events of e have subparts of x as their themes. On the other hand, sub-events of x filling still have x as their theme; it is just x's degree of fullness that differs; Thus, *fill* is the perfect example of the sort of mislabeled incremental theme discussed by Hay et al. (1999). The participant bearing a homomorphic relation to the event really is a degreeable property of the theme, not the theme itself. In contrast the facts with *cover* suggest that it is ambiguous, and has both a mereological state function and degree-taking state function, and that on the former reading the theme is incremental and on the latter it is not.

Contrast the case of *eat*:

(52) a. ? Fred ate the candy bar more than Sue.
b. Fred ate more of the candy bar than Sue.

c. * Fred half ate the candy bar.
d. a half-eaten candy bar

(52a) has a somewhat marginal frequentative interpretation, but no interpetation limited to a single occasion of eating; in particular, (52a) does not have an interpretation requiring Fred to eat a greater volume of candy bar, as (b) does. This suggests the *eat is* an incremental theme verb in the sense of Krifka (1998), that the themes of subevents of candy bar-eating are parts of the candy bar, and that this is a lexical difference distinguishing *eat* from *fill*. This is consistent with the unacceptability of (c), while the acceptability of (d), the sort of example discussed extensively in Kennedy and McNally (1999), suggests that a mapping onto a linear scale has occurred in deriving the adjective.

Since all scalar adjectives allow comparatives, it appears that a necessary condition for scalar adjectives is that their state functions be scalar; The derived verb *fill* seems to inherit that requirement, while deverbal adjectives are coerced to obey this requirement. What I am suggesting is that for at least some non-derived verbs like *cover* and non-scalar extent verbs like *extend*, that condition is relaxed,

I have assumed throughout this paper thatthere are two kinds of change, change by parts and change by degrees. Here I have argued that the homomorphic participants are different for the two kinds of change. When the theme is incremental, sub-events have subparts of the theme as theme; when the degree is incremental, subevents have the entire theme as theme. Notice you can't mix these two kinds of account. There is a sub-event of the candy bar's being eaten in which the candy bar is half-eaten, but if the theme of that sub-event is half the candy bar (Krifka's picture), then the predicate that describes that event is *eat*, not *half-eat*. On the other hand if the predicate is *half-eat* then the theme must be the whole candy bar; it isn't half the candy bar that's half eaten. And the fact that

(53) a. The fog covered more of the valley.
b. Fog covered the valley more.

are both equally natural descriptions of the same event really suggests that two state functions with distinct incremental participants are required.

Without attempting to do this complex topic complete justice, I point to two further motivations why both kinds of state functions might be motivated:

1. A lot of what's at issue in the body of work cited above is how to change from adjectival scalar semantics to verbal scalar semantics:

   (54) a. He opened the door *a bit*.
   b. The door was *a bit* open.

The same modifier (italicized) occurs in both (54a) and (54b). This then provides an argument for extending degree analyses from adjectives to verbs. A nuumber of languages (including English) offer good evidence that the extension should continue into the domain of parts. There are a number of modifiers that apply to both degrees and parts:

(55) a. *A bit* of bacon remained.
b. He was *a bit* tired.
c. The exercise tired him *a bit*.
d. *A lot* of bacon remained.
e. He was a *a lot* more tired.
f. The exercise tired him *a lot*.
g. *A smidge* of bacon remained.
h. He was a *a smidge* more tired.
i. The exercise tired him *a smidge*
j. He was *a little* tired.
k. *A little* sausage remained.

Similar sets of examples can be constructed in French (*un peu fatigué*, 'a little tired', *un peu de jambon*, 'a little ham') and Polish (*trochę zmęczony*, 'a little tired', *trochę kiełbasy*, 'a little sausage')

2. Whether or not the particular analysis of *cover/* advanced here is correct, adverbs like *gradually* may be licensed by quantification over parts:

(56) a. # He gradually bought the newspaper.
b. He gradually bought the entire development.

If, as we have assumed, *gradually* requires an increasing state-function with a mereological domain, that domain can be supplied by quantification over parts. Thus *gradually* becomes another entry in our list of modifiers sensitive to both degrees and mereologies.

Summing up, mereological approaches to aspect (Krifka 1989, Krifka 1992, Krifka 1998, Pinon 1994a, Pinon 1994b, Ramchand 1997, Filip 1999) have sometimes been held up as competitors to a scale-based account. My main point is that a little mereology provides an essential complement to degrees and can capture important lexical distinctions like that between *fill* and *eat*. The supervening concept of a mereology with remainders thus provides the central structural feature of an account of gradual change. Whether the change function returns degrees or mereological sums, something like the following will be true and an account of aspectual properties will follow:[19]

(57) **Change bounding**

$$e' \leq e \Longrightarrow \Delta_e(e') \leq \Delta_e(e)$$

This requirement is too strong, but correctly expresses a kind of default truth. Understood as a requirement extending over both change by degrees and change by parts, this basically says that for verbs of gradual change, Δ_e must be increasing in moving from subevents to larger events.

6 A conclusion

The results of this paper are essentially the following:

(a) Event and extent ambiguities can be accounted for with state functions whose domains may be either temporal or spatial indices;

(b) Predicates with state functions that allow spatial indices are also predicates that allow spatial paths. Spatial indices require an oriented axis of the sort used elsewhere in the language of space, and spatial paths are the primary device for describing and evoking such axes;

(c) This establishes a domain of predicates with *spatial aspect*. Spatial aspect varies just as temporal aspect does. There are spatial operators that map spatial states to spatial accomplishments/activities;

(d) This has led to the proposal of a general characterization of verbs of gradual change: All verbs of gradual change have non-trivial change functions with mereologies as their ranges. This can be viewed simply as generalization of the degree hypothesis of HKL.

(e) There are two kinds of gradual change, change by degree and change by parts, with corresponding changes in the range of the state function. The ranges of the state functions of verbs of gradual change must be mereologies with remainders.

There is a natural way of generalizing the view of state functions explored here, which helps relate operators like INCREASE which operate over ordered domains with mereological structure, to operators like Dowty's 1979 BECOME operator, which accounts naturally for non-gradual change.

The point is this: All states, activities, accomplishments, and achievements may be viewed as having state-functions which obey the mereology axioms minus the Remainder Principle. To take this step would require admitting into the fold **Boolean** state functions for verbs of non-gradual change such as *solve, prove, give*, etcetera. The Boolean state function for *solve*, for example, is a function of time which returns $\textsc{false}$ for the entire temporal span of a solving event until the theme is solved. A Boolean domain is still ordered, and if we adopt

$$\textsc{false} < \textsc{true}$$

then the transition from non state to state will be an increase. Such a domain does not obey the Remainder Theorem, although the function does obey the non-triviality requirement, so this would be the essential structural difference between gradual change and non-gradual change. On this view, INCREASE and BECOME fall on a natural continuum.

This also opens up another possibility. The fact that there is no graduality with extent *cover*, no verb with the semantics

$$\textsc{increase}_{\mathrm{S}}(\mathrm{cover}_{\mathrm{S}})$$

as we saw in (30), may also be explained simply by saying there is no predicate with the meaning:

(58) $\mathrm{cover}_{\mathrm{S}}$.

That is, we could simply say that the state function in (58) is not a legitimate state function for a natural language predicate to have. Thus there can be no stative adjective or verb which has (58) as its meaning because its range is trivial. Such functions arise only as steps in the definition of legitimate state functions like:

$$\mathrm{cover}_{\mathrm{T}}^{\mathrm{S}},$$

which is nontrivial, and legitimate change functions like:

$$\textsc{increase}_{\mathrm{T}}(\mathrm{cover}_{\mathrm{T}}^{\mathrm{S}}).$$

Appendix

Definitions of path operators

The domain of any path function π is that set of points on the axis S that fall within e:

$$\mathbf{path}_{\mathrm{I}}(e){=}\pi \quad \text{only if} \quad \pi : [\textsc{start}_{\mathrm{I}}(e),\ \textsc{end}_{\mathrm{I}}(e)] \rightarrow \mathrm{Locations}$$

Loc is a function returning the entire spatial region covered by a path function, defined as:

$$\mathrm{Loc}(\pi) = \bigsqcup_{s \in \mathrm{Dom}(\pi)} \pi(s)$$

Temporal and spatial paths are defined by means of a location function AT, which returns the location of its argument at a time t:

(a)	**Spatial**	$\textbf{path}_{\mathrm{S}}(e)(s)$	$=$	$\textsc{at}(\mathrm{theme}(e), \mathcal{T}(e)) \bigsqcap \mathrm{plane}(s, \mathrm{S})$
(b)	**Temporal**	$\textbf{path}_{\mathrm{T}}(e)(t)$	$=$	$\textsc{at}(\mathrm{theme}(e), t)$
(c)	**Temporal Coercion**	$\textbf{path}^{\mathrm{S}}_{\mathrm{T}}(e)(t)$	$=$	$\textsc{at}(\mathrm{theme}(e), t) \bigsqcap \mathrm{Loc}(\mathrm{path}_{\mathrm{S}}(e))$

A key property is that path always returns a region of space, whether temporal or spatial; (a) Spatial path always returns the location of the figure at slice s within the temporal bounds of e $(\mathcal{T}(e))$; (b) Temporal path always returns the location of the figure at the relevant time within the spatial trace of of e $(\mathcal{S}(e))$.

All the aspectual differences between spatially and temporally indexed predicates then follow because temporal paths must overlap at successive moments of times, but spatial paths cannot overlap at successive spatial indices. The temporal coercion cases behave like temporal paths, only restricted by the the spatial path of the event.

I also assume a family of event-independent path functions incorporating spatial relations other than AT. These will be used, among other things, for the semantics of path prepositions like *into* and *onto*. As an example, the definition of on_{S} follows:

$$\textsc{on}_{\mathrm{S}}(x)(t)(s) = \textsc{on}(x, t) \bigsqcap \mathrm{plane}(s, \mathrm{S})$$

The function ON is a spatial function returning the supporting surface region of its argument at a time t. Thus for each spatial index s, $\textsc{on}_{\mathrm{S}}(e)(s)$ returns the slice of the theme's supporting surface at s.

Mereologies

We take a mereology to be a join-semi-lattice in which the Remainder Principle is satisfied. The following definitions, in slightly modified form, are from Krifka (1998:199):

(59) $\mathrm{P} = \langle \mathrm{U}_{\mathrm{P}},\ \oplus_{\mathrm{P}} \rangle$ is a **part-structure** iff

(a) $\mathrm{U_P}$ is a set of entities;
(b) $\oplus_{\mathrm{P}}$, the **sum (join) operation**, is a function from $\mathrm{U_P} \times \mathrm{U_P}$ to $\mathrm{U_P}$ that is idempotent, commutative, and associative.

From $\oplus_{\mathrm{P}}$ we may define 3 relations:

(60) (a) $\leq_{\mathrm{P}}$, the **part-of relation**, defined as $\forall x, y \in \mathrm{U_P}[x \leq_{\mathrm{P}} y \leftrightarrow x \oplus_{\mathrm{P}} y = y]$
(b) $<_{\mathrm{P}}$, the **proper part-of relation** , defined as $\forall x, y \in \mathrm{U_P}[x <_{\mathrm{P}} y \leftrightarrow x \leq_{\mathrm{P}} y \wedge x \neq y]$
(c) $\otimes_{\mathrm{P}}$, the **overlap relation**, defined as $\forall x, y \in \mathrm{U_P}[x \otimes_{\mathrm{P}} y \leftrightarrow \exists z \in \mathrm{P}[z \leq_{\mathrm{P}} y \wedge z \leq_{\mathrm{P}} x]\,]$

It is easy to show that $\leq$ is reflexive, transitive, and anti-symmetric. From the fact that $\oplus$ is idempotent, commutative, and associative it follows that $x \oplus y$ is an upper bound on x and y:

$$\begin{aligned} x \oplus (x \oplus y) &= (x \oplus x) \oplus y \\ &= x \oplus y \\ y \oplus (x \oplus y) &= y \oplus (y \oplus x) \\ &= (y \oplus y) \oplus x \\ &= y \oplus x \\ &= x \oplus y \end{aligned}$$

It is easy to show that $x \oplus y$ is a least upper bound as well. So this shows a part-structure is a join semi-lattice with ordering relation $\leq$ and join operation $\oplus$. Some authors (Pinon 1994a) simply use the term mereology to mean a part structure with the definitions in (60) added.

With Krifka, we use **mereology** to mean a part structure in which any ordered pair of ordered elements, x and y, has a unique *relative complement* r. That additional requirement is called the Remainder Principle:

Remainder (relative complement) principle:

$$\forall x, y \in \mathrm{U_P}[x <_{\mathrm{P}} y \rightarrow \exists! r[\neg[r \otimes x] \wedge x \oplus z = y]]$$

As Krifka points out, structures that respect the Remainder Principle exclude bottom elements (elements that less than all others), because everything overlaps with bottom, and the remainder axiom requires every non-maximal element have at least one non overlapping element. Thus sets of degrees must exclude 0. In order to satisfy these axioms, a set of degrees must also be closed under the difference operation.

Endnotes

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[1] Not quite what he says. Fix this up.

[2] Extent readings for degree achievements, however, are not restricted to clearly spatial adjectives. Any degreeable adjective whose degree can change with respect to space while time is held constant can yield an extent reading:

(i) The sky pinkened in the east.

We assume then, that a reference axis may be supplied for any such adjective.

[3] See Gawron (2006) for discussion.

[4]The modification is that adjective meanings have a state argument, and a Neo-Davidsonian style of breaking out roles has been used.

[5] $\mathcal{T}(e)$ is Krifka's (1998) spatiotemporal trace function.

[6] The idea of representing paths through the use of functions from events to times to locations is anticipated in Verkuyl 1978, Verkuyl 1993. This model of path is also consistent with axioms of Krifka (1998).

[7] See the appendix for the full definitions of temporal and spatial path functions.

[8] We assume a PP like *from Boston* denotes a property of paths:

$$\begin{aligned} [\![\text{from Boston}_{\mathrm{S}}]\!] &= \lambda\pi[\pi(\textsc{start}_{\mathrm{S}}(e)) \text{ overlaps Boston}] \\ &= \text{a property true of a path if the path evaluated at minimal member of its domain overlaps Boston} \end{aligned}$$

The bracketed $[v : r]$ in (19) designates a property of path functions true if they begin at v and end at r. At the minimal index of the event the fog must overlap the floor and at the maximal index the ridge.

[9]See the appendix for the definition.

[10] It is a nice feature of INCREASE and temporally indexed paths, confirming the utility of both, that they combine to give the rather unexpected semantics of spreading motion.

[11] The simplest measure is simply to count the number of such direction changes; that is, the range of ZIGZAGGY can be integers, as such suggested by such examples as *The road zigzagged 5 times en route to the summit*. This might also be a reasonable analysis of semelfactive verbs such as *jump* and *flash*: they are verbs of gradual change whose state functions return integers counting the number of times some basic "step" is iterated.

[12] We are not assuming a temporal spatial asymmetry here; we are focusing on the case that is relevant for developing the examples of Section 1. Note that width claims can be made over temporal intervals as well:

(i) The flood channel was 3 feet wide from 3 to 4 o'clock.

This in fact does seem to have a reading completely parallel to (24), that there is a contextually available point in space at which the channel was 3 feet wide over the given temporal interval.

[13] I discuss the somewhat different case of *fill* in Section 5

[14] the definition is given in the appendix.

[15] The description and terminology in this section follows many of the ideas of Jackendoff (1996), though I have departed somewhat from his usage, in not allowing two-dimensional predicates. Jackendoff allows what he calls two-dimensional predicates.

[16] The idea of accounting for such ambiguities by associating verbs with a degreeable property is also the centerpiece of Zucchi (1998). He accounts for the aspectual amibuity of verbs like *bake* by associated them with degreable result properties, but does not extend the account to degree achievements. In fact he explicitly draws the line at extending the account to *grow*, but I am not entirely clear why.

[17]HKL use the term difference argument, but this term does not seem to be appropriate for cases *zigzag*, where the INCREASE is not involved

[18] A possible line of defense I will not pursue here is to continue to defend a mereological state function for *full* and to try to assimilate both examples in (48) to the class of comparatives Kennedy (1999), citing Bierwisch (1996:220), calls Comparison of Deviation (COD) constructions:

(i) The Red Sox will scrutinized as closely as the Orioles to see whether they are any more legitimate than the Orioles are fraudulent.

Here clearly comparison is being performed between two different scales and the right interpretation is that what is being compared is extent of deviation from the standard; hence the name, Comparison of Deviation. The application to (48) is as follows: the comparison is between two distinct mereological orderings, the standards in the two cases are the maximal regions inside the respective glasses, and the comparison is of the deviation from those two standards.

[19] I have omitted from this discussion any mention of what is sometimes thought of as a central part of the program of a mereological account of aspect, homomorphisms from events to participants. I have omitted it this because this issue doesn't separate a degree-account from a mereological account. Both domains are ordered and in both cases I believe something like a homomorphism will be required. Assuming a change function Δ on events, and assuming that Δ is a **bounding change function**, then (57) always follows, whether the range of Δ is degrees or a mereology. For example, assume a degree-account of aspect and something like Krifka's 1989 account of temporal frame adverbials like *for 5 minutes*; what matters is whether the VP event property is quantized or not. What a degree account claims is that event properties of e will be quantized if they are equivalent to a property conjunction containing some quantized property of the degree of change $\Delta(e)$. But this means something like (57) is true. That is, if temporal modifiers are properties of events, and if quantization is the relevant property of event properties, then an explicit formal account needs to be be able to derive some order-preserving mapping from e and e' to $\Delta(e)$ and $\Delta(e')$ in order for quantization of the $\Delta(e)$ property to predict quantization of the event property (and therefore the distribution of *for*-adverbials). The reason I keep saying "something like" a homomorphism instead of a homomorphism is that the degree account has the same kinds of problems Krifka (1998) raises for the mereological account in the case of what he calls "strict incrementality". The progress of events even with fairly prototypical incremental themes does not have to monotonically track the part of relation. Part of a book can be reread before completing it. Surely no one who has ever actually written a paper believes in the strict incrementality of the verb *write*. Similarly, parts of a sonata can be forgotten and relearned in the process of learning, in which case we have $e \leq e'$ but $\Theta(e') \leq \Theta(e)$. For degree achievements, clearly when a room cools 5 degrees in 5 minutes, it can be true that, for brief subintervals of that time, the temperature actually rose. Thus for both accounts, the $\Longrightarrow$ in (57) has to mean something like default implication or "implies in minimal models."

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