Modality tutorial

http://www-rohan.sdsu.edu/~gawron/optimality

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Overview

Outline

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Goals

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- Explain the truth conditions of some complicated sentences in terms of the truth conditions of simpler sentences.
- Deal with some of the linguistic variety of modality: Auxiliary verbs, adjectives, adverbs, Conditional sentences
- Understand a bit better what possible worlds are and what they do for us.

Outline

Examples: Necessity

- Necessarily, bachelors are unmarried.
- It is necessarily the case that a bachelor is unmarried.
- It is always the case that a bachelor is unmarried.
- A bachelor must be unmarried.
- A bachelor has to be unmarried.
- # A bachelor is sure to be unmarried.
- A bachelor is unmarried. [Generic reading]

Examples: Possibility

- Possibly, there will be an earthquake tomorrow.
- A triangle may have 3 sides of different lengths.
- It is possible for a man to be older than his own uncle.
- A right triangle may have 3 sides of equal length. [True?]

Further/covert Examples I

- Al Gore almost won the 2000 election. (If not for a few hanging chads, Al Gore would would have won the 2000 election).
- North Carolina was almost the 2016 NCAA national basketball champion.
- North Carolina came within an eyelash of winning the 2016 NCAA national championship in basketball.
- John was writing his dissertation when he died. (cf. John was watching when Bill entered the room.)
- Adjectives: possible, breakable, readable, edible, soluble, flammable, inconsolable, unforgettable, combustible. Do *flammable* (and *combustible*) just mean *able to be burned*? Does *readable* just mean *possible to read*.
- *easy to*: *That book is easy to read* = *That book is readable*.

Further/covert Examples II

- English *-able* is like German *-lich* and *-bar*, so *unvergesslich* is *unforgettable* and *löslich* is *inconsolable*. But not consider *sterb-lich* (= *martal*), that is, capable of dying, *erblich* (= *hereditary*), capable of being inherited. Doesn't German have a point?
- Also *probably*, *to be able to*, *to be in the position to*
- Doesn't fragile mean easily damaged. Does *smart* mean *capable of solving problems*?

Summary

- Possibility and necessity based modality
- Grades of modality (possible, likely, probable)
- Lexically incorporated modality *breakable, edible, flammable*
- Covert modalit

Logical analysis

a.	Necessarily, bachelors are unmarried	$\Box$unmarried(b)
b.	Squares must be 4-sided.	$\Box$4-sided(s)
c.	Possibly it will rain	$\Diamond$rain()
d.	Right triangles may not have 3 sides of equal length	$\sim \Diamond$3-equal-sides(rt)
e.	Necessarily, right triangles do not have 3 sides of equal length	$\Box \sim$ 3-equal-sides(rt)

Note that it's wrong to write the following for (d):

$$\Diamond \sim \text{3-equal-sides(rt)}$$

This translates: *it is possible that right triangles do not have three equal sides*.

A logical relationship: Duals

1. $\Box \sim p \iff \sim \Diamond p$
2. $\sim \sim \Diamond p \iff \sim\sim \Diamond p$
3. $\sim \Box \sim p \iff \sim\sim \Diamond p$
4. $\sim \Box \sim p \iff \Diamond p$

$\Box \sim q \iff \sim \Diamond q$ Set q to $\sim p$

$\Box \sim\sim p \iff \sim \Diamond \sim p$

$\Box p \iff \sim \Diamond \sim p$

Outline

Logical

p is possible/necessary relative to **all** possible worlds

- A dog may have 3 legs.
- A triangle must have 3 legs.
- It must either be raining or not raining.
- Mitt Romney might have won the 2012 election (if not for that foolish 47% gaffe).
- Napoleon might have won at Waterloo (if not for the dysentery afflicting his troops).

Epistemic

Given what we know (for all we know), p.

- The murderer may have entered through the library.
- The murderer must be right-handed.
- The dinosaurs must have died out suddenly.
- John may go to Cozumel. (possibility in the future)
- John must be in Dubai by now. (necessity in the present)
- Kennedy might not have been shot by Oswald.

Logical vs. epistemic

- Logical possibilities include those that are contrary to fact.

> *She might have fallen down the cliff. Thank god her rescue harness held.*

- Epistemic possibilities do NOT include possibilities that are contrary to fact.

> *She might have fallen down the cliff. We're still waiting to hear from the rescue party.*

Deontic

p is possibile/necessary in **perfectly obedient** worlds.

- A zombie must be clean and courteous.
- You can smoke only in the designated areas.
- You may have a cookie.
- John has to take Ling 525.

The big picture

	Necessary	Possible
Logical	A bachelor must be unmarried	A triangle may have unequal sides
Epistemic	The murderer must be right-handed	The murderer may have entered here.
Deontic	A zombie must be courteous.	You may smoke here.

Ambiguity

- Kennedy might not have been shot by Oswald.
- Sue may not go to the movies.
- John must be in class.
- A woman might have written the *Odyssey*.

Negation

- The murderer could not have entered through the window.
- The murderer might not have entered through the window.
- John must not be lying.
- John doesn't have to to be lying.

Logical Representations

The murderer could not have entered.	$\sim \Diamond$enter(m)
The murderer might not have entered.	$\Diamond \sim$ enter(m)
John doesn't have to be lying.	$\sim \Box$lie(j)
John must not be lying.	$\Box \sim$ lie(j)

Auxiliaries

- Modal auxiliaries

  (1) a. John may go.[at least two readings]
  b. John must go. [at least two readings]
  c. John can go. [at least two readings]
  d. John should go.
  e. John might go.

- Non "modal" concepts. For now, we set aside tense readings, ability readings

  (2) a. John will go.
  b. John can dance. (= *John is able to dance*)

Semi-modals

Other "modal" verbs (sometimes called "semi-modals")

(3) a. John has to go.

b. John ought to go.

c. John need go only if he runs out of money.

Don;t invert in yes-no questions (or marginally so, for many speakers)

Outline

Necessity as universal quantification

1. $\Box p$ means: However things might actually turn out to be (or to have been), p is true.
2. We have to talk about p being true or false depending on which way things turn out.
3. What does *a way things turn out* mean?
4. Since a **world** is determined by an assignment of truth values to all atomic statements, we can use a world to capture the idea of *a way things turn out*.
5. $\Box p$ means: in all possible worlds p has to turn out to be true.

Logical Necessity: truth in all possible worlds (W)

1. One issue: If we can just assign truth values to an atomic statement like p willy nilly, how does any p turn out to be necessarily true?
2. Answer: We have postulates (**Meaning postulates**) the truth assignments in all worlds must be consistent with:

$$\forall x\,[\,\text{bachelor}(x) \rightarrow \text{unmarried}(x)\,]$$

3. So we are assigning truth values to atomic statements *in accordance with our knowledge of the entailment properties of English predicates*.
4. Therefore,

$\Box\,[\,\forall x\,[\,\text{bachelor}(x) \rightarrow \text{unmarried}(x)\,]\,]$ or, equivalently,
$\forall w \in \text{W}\,[\,\forall x\,[\,\text{bachelor}(x) \rightarrow \text{unmarried}(x)\,]$ is true in $w\,]$

What about **epistemic** necessity?

Epistemic necessity is truth in all worlds consistent with what we know (all propositions we **know** to be true are true in these worlds). We call the set of worlds consistent with what we know **the epistemically possible** worlds. We write the set as **E**.

Smith must be the murderer
$\Box_{\mathrm{E}}$ [murdered(smith)]] or, equivalently,
$\forall w \in \mathrm{E}$ [murdered(smith) is true in w]

When we're being careful, we'll relativize E to a world w (the set of worlds consistent with what is known in w) or to a community or to just one person.

What about **deontic** necessity?

Deontic necessity is truth in all worlds that are obedient to the rules, or to what is required, or commanded. We call this set of obedient worlds **the perfectly obedient** worlds. We write the set as **PO**.

Cinderella must return by midnight
$\Box_{\mathrm{E}}$ [return-by-midnight(Cinderella)]] or, equivalently,
$\forall w \in \mathrm{PO}$ [return-by-midnight(Cinderella) is true in w]

When we're being careful, we'll relativize PO to a world w (the set of worlds perfectly obedient to what is required in w) or to a community.

Veridicality

Non-veridical contexts

Non-veridical contexts are linguistic contexts in which normal existence entaliments fail. Model contexts are non-veridical.

1. John married an elf. $\Rightarrow$ An elf exists.
2. John may marry an elf. $\not\Rightarrow$ An elf exists.

Logical notation

1. $\exists x\,[\,\mathrm{elf}(x)\,\&\,\mathrm{marry}(\mathrm{j},\,x)\,]\Rightarrow\exists x\,\mathrm{elf}(x)$
2. $\Diamond\exists x\,[\,\mathrm{elf}(x)\,\&\,\mathrm{marry}(\mathrm{j},\,x)\,]\not\Rightarrow\exists x\,\mathrm{elf}(x)$

The consequences

$$\Box p \Longleftrightarrow \forall w[p \text{ is true in } w]$$
$$\sim \Box \sim p \Longleftrightarrow \sim \forall w[p \text{ is not true in } w]$$
$$\sim \Box \sim p \Longleftrightarrow \sim \forall w \sim [p \text{ is true in } w]$$
$$\Diamond p \Longleftrightarrow \sim \forall w \sim [p \text{ is true in } w]$$
$$\Diamond p \Longleftrightarrow \exists w[p \text{ is true in } w]$$

Outline

Truth definitions for English sentences

1. 'Necessarily, a bachelor is unmarried' is true iff For all possible worlds $w \in$ W 'a bachelor is unmarried' is true in w
2. 'A child could have invented the moustrap' is true iff There exists some possible world $w \in$ W and 'A child invented the mousetrap' is true in w
3. 'The lake is sure to freeze tonight' is true iff for all possible worlds $w \in$ E 'The lake freezes tonight' is true in w
4. 'Villagers goats may graze on the green' is true iff for all possible worlds $w_{po} \in$ PO, 'Villagers goats graze on the green' is true in w_{po}.

Truth definitions for $\Box$ and $\Diamond$

1. $\Box p$ is true iff For all possible worlds $w \in \mathrm{W}$ p is true in w.
2. $\Diamond p$ is true iff There exists some possible world $w \in \mathrm{W}$ such that p is true in w.
3. $\Box_{\mathrm{E}} p$ is true iff for all possible worlds $w \in \mathrm{E}$ p is true in w

Ambiguity

Modality assignment I

1. Give truth definitions, giving more than one truth definition with ambiguous examples.
   1. You may not enter the den.
   2. You are not allowed in the den.
   3. You must report to the principal's office.
   4. She might have been arrested.
2. Give logical representations, using predicate logic and $\Box$ and $\Diamond$. For example, 'John must not be married.' would be:

$$\Box \sim \text{married(j)}$$

   1. John might not have come early.
   2. The president could not have been in the oval office.