Logical Translations

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1 Examples

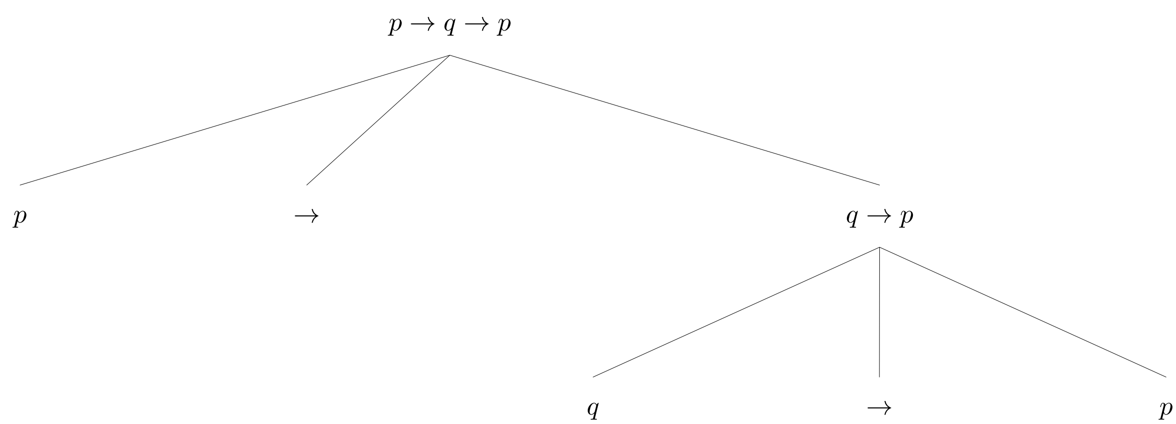

Figure 1: Logic tree for $p \to (q \to p)$

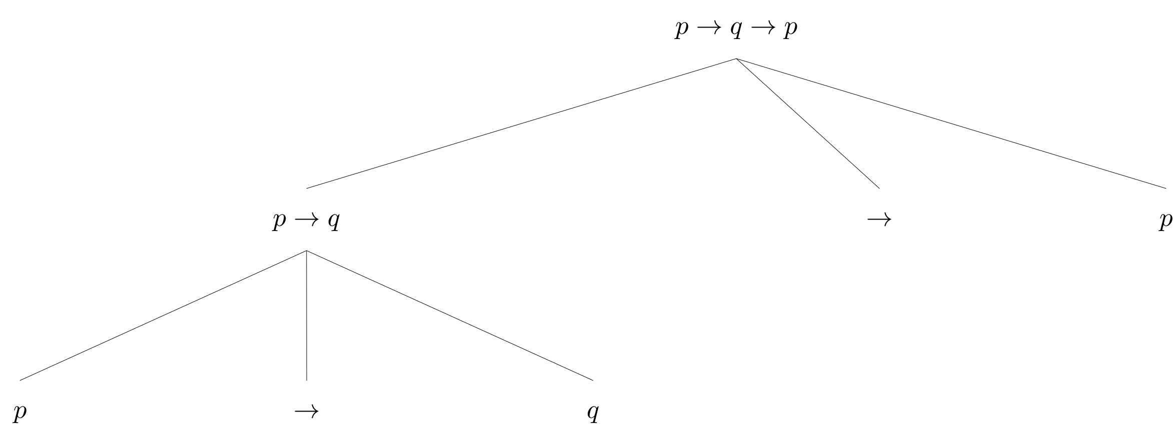

Figure 2: Logic tree for $(p \to q) \to p$

	p	q	$p \to q$
(a)	T	T	T
(b)	T	F	F
(c)	F	T	T
(d)	F	F	T

Truth table for $p \to q$ Truth c

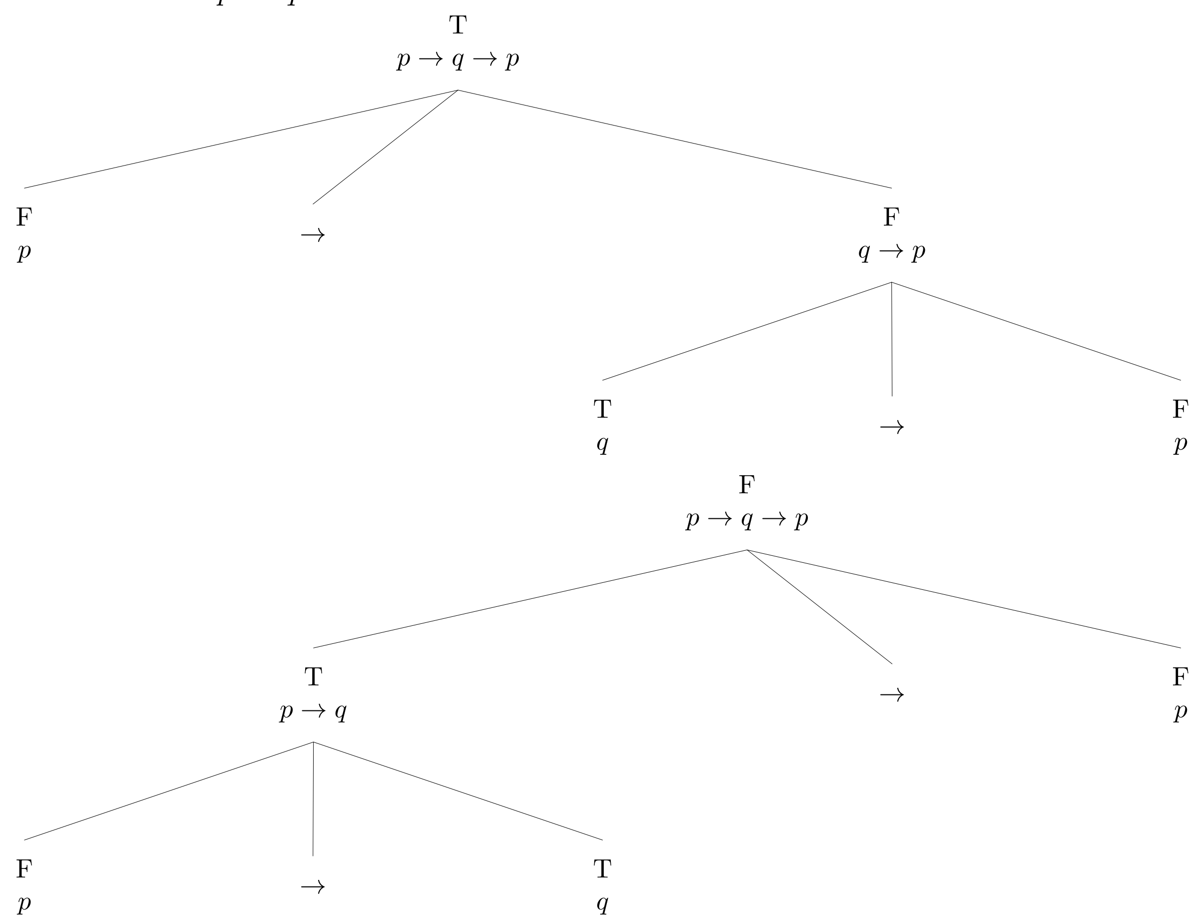

Figure 3: Logic trees for $p \to (q \to p)$ and $(p \to q) \to p$ in w_3

	p	q
w_1	T	T
w_2	T	F
w_3	F	T
w_4	F	F

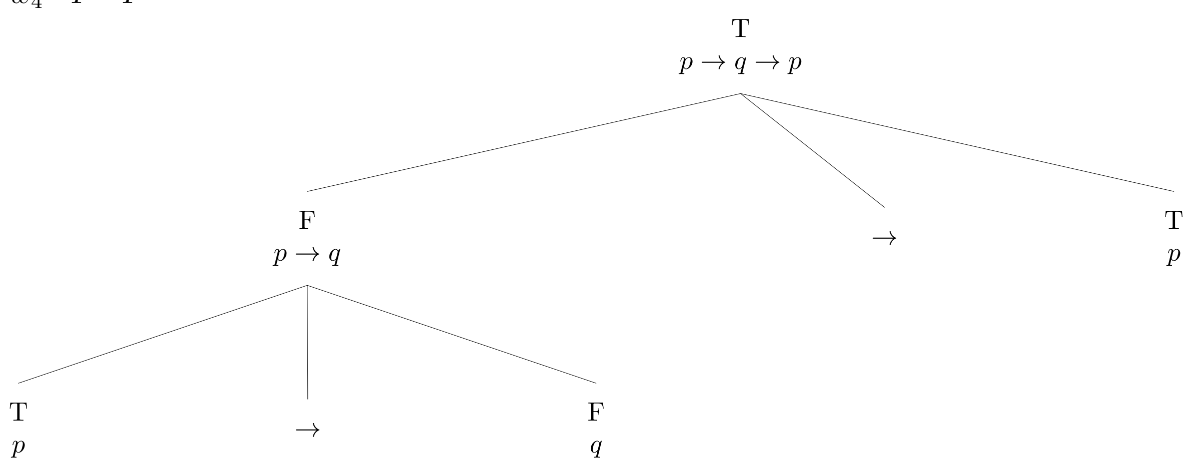

Figure 4: Logic tree for and $(p \to q) \to p$ in w_2

2 Truth tables

Figure 3 shows the logic tree for $(p \to q) \to p$ in w_3 and Figure 4 shows the logic tree for $(p \to q) \to p$ in w_2. This means we know the truth values of $(p \to q) \to p$ when p is F and q is T (w_3) and we also know the truth value of $(p \to q) \to p$ when p is T and q is F (w_2). So we know two of the four lines for the truth table for $(p \to q) \to p$.

	p	q	$p \to (q \to p)$
(a)	T	T	
(b)	T	F	T
(c)	F	T	T
(d)	F	F	

A truth table computes the truth values for an expression in all possible worlds. But we dont really need to look at all possible worlds to do that. We just need to look at all possible ways of assigning truth values to the sentential letters in the expressions (p and q in this case). Since we have 2 letters, there are 4 different ways of assigning the values T and F and so the truth table has 4 rows. If there were 3 letters, there would 8 rows. In general, if there are n letters, a truth table would have 2^n rows, so they can get quite long.

To finish our truth table, we need to deal with the cases where p and q have the same truth values, that is where p is T and q is T, row (a), and with the case p is F and q is F, row (d).

	p	q
w_1	T	T
w_2	T	F
w_3	F	T
w_4	F	F

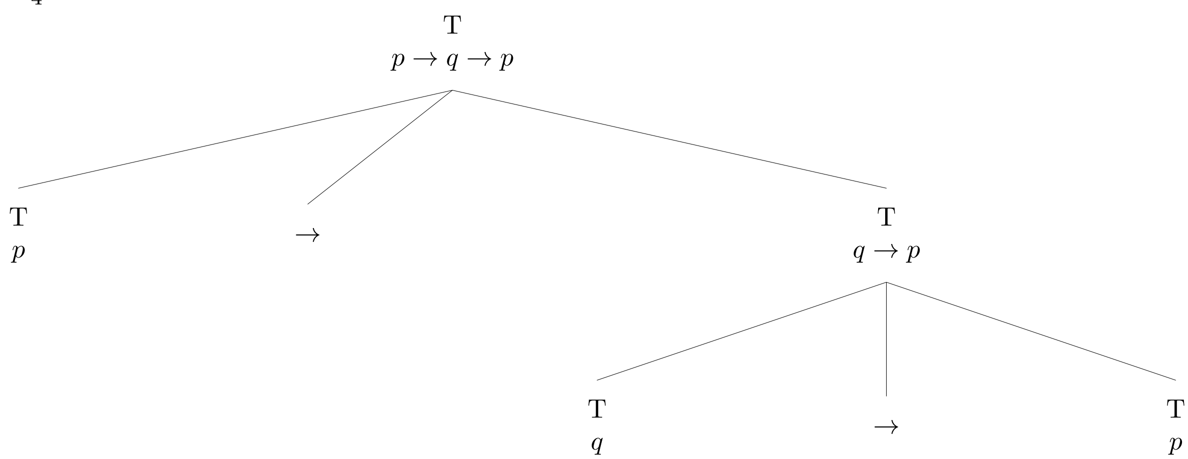

Figure 5: Logic tree for and $p \to (q \to p)$, row (a) of truth table

	p	q
w_1	T	T
w_2	T	F
w_3	F	T
w_4	F	F

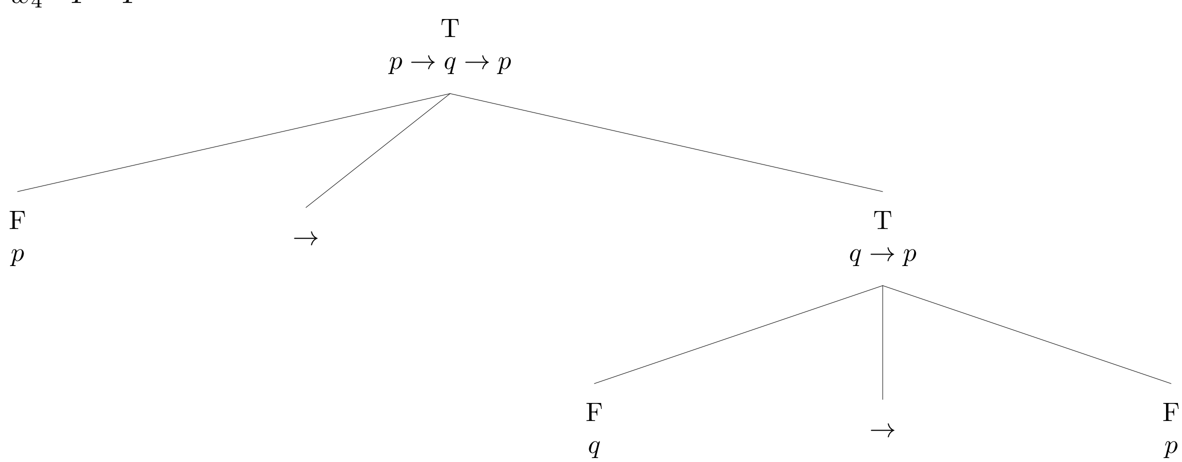

Figure 6: Logic tree for and $p \to (q \to p)$, row (d) of truth table

Now the completed truth table.

	p	q	$p \to (q \to p)$
(a)	T	T	T
(b)	T	F	T
(c)	F	T	T
(d)	F	F	T

Task: Complete the truth table for $(p \to q) \to p$. Line (c) has already been done for you. (Figure 3)

	p	q	$(p \to q) \to p$
(a)	T	T	
(b)	T	F	
(c)	F	T	F
(d)	F	F	

3 In class exercises

Truth tables: Do complete truth tables for all of the following expressions

In the following exercises, assume the following facts about worlds w_1, w_2, w_3 and w_4.

	p	q	r
w_1	T	T	T
w_2	T	F	F
w_3	F	T	F
w_4	F	F	T

Assume the standard truth tables for & and $\vee$:

p	q	$p \mathbin{\&} q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

1. Draw the logic tree for

$$(p \,\&\, q) \vee r$$

in w_3.

2. Draw the logic tree for

$$(p \,\&\, q) \vee r$$

in w_4.

3. Let:

$$p = \text{John sleeps}$$
$$q = \text{Mary sleeps}$$

Do the logic tree for “John or Mary sleeps” in w_2.

4. Consider “Neither John nor Mary sleeps”. Assume

$$p = \text{John sleeps}$$
$$q = \text{Mary sleeps}$$

This is a two-part problem.

Part A: Using your linguistic intuitions, evaluate the TRUTH of this sentence in worlds w_1 through w_4, and fill in the following table with the results, assuming p is *John sleeps* and q is *Mary sleeps*. I have done the first row for you.

	p	q	Neither p nor q
w_1	T	T	F
w_2	T	F	
w_3	F	T	
w_4	F	F	

The first row reflects my intuition that “Neither John nor Mary sleeps” is FALSE in a world where “John sleeps” (p) is TRUE and “Mary sleeps” (q) is TRUE.

Part B: From among the following alternatives, you must choose a logical translation for *Neither John nor Mary sleeps*:

$$\begin{array}{ll} a. & \sim p \vee q \\ b. & \sim (p \vee q) \\ c. & \sim p \;\&\; \sim q \\ d. & \sim p \;\&\; q \\ e. & p \vee q \\ f. & p \;\&\; q \\ g. & \sim (p \;\&\; q) \end{array}$$

To show that your choice is valid, you must show the entire truth table for your choice.