

1 Introduction

We introduce the connectives of **statement logic**.

We introduce predicates. And a very simple semantics for them.

We introduce important **semantic relations** among sentences: entailment, contraries, contradictories. We introduce related properties of single sentences: logical truth (tautology), contradiction.

2 Truth-Functional Connectives

2.1 And

Consider our extension rule for 'and'

$\llbracket A \text{ and } B \rrbracket = \text{True}$ if and only if $\llbracket A \rrbracket = \text{True}$ and $\llbracket B \rrbracket = \text{True}$.

	A	B	$A \text{ and } B$
(a)	T	T	T
(b)	T	F	F
(c)	F	T	F
(d)	F	F	F

This says the same as the rule for $\&$ in the text book:

	p	q	$p \& q$
(a)	T	T	T
(b)	T	F	F
(c)	F	T	F
(d)	F	F	F

Idea: Use $\&$ as the **logical translation** for *and*:

- (1) a. Abraham Lincoln was elected in 1860 and he was re-elected in 1864.

- b. John picked up the apple and he ate it.
- c. ? John ate the apple and he picked it up. [temporal order, pragmatics?]
- d. You take one more step and I'll shoot. [= If you take one more step, I'll shoot]

2.2 Or

	p	q	$p \vee q$
(a)	T	T	T
(b)	T	F	T
(c)	F	T	T
(d)	F	F	F

- (2) a. Either he rented either a mid-size car or he rented an economy car. [$p \vee q$ translation claims: If in fact he rented both, this is still true]
- b. Either there's no bathroom in this house or it's on the second floor. [In fact both statement can't be simultaneously true.]
- c. You can have either the white one or the red one. [intended meaning: but not both]

Exclusive or

	p	q	$p \bar{\vee} q$
(a)	T	T	F
(b)	T	F	T
(c)	F	T	T
(d)	F	F	F

Predictions of exclusive-or analysis:

- (3) Maria is very smart, or she is very hard-working.

Suppose Maria is both.

2.3 Material implication

Material implication is the name we'll use for \rightarrow .

	p	q	$p \rightarrow q$
(a)	T	T	T
(b)	T	F	F
(c)	F	T	T
(d)	F	F	T

- (4) a. If Alice wins a fellowship, she will finish her thesis.
 b. Antecedent: Alice (will win)/wins a fellowship.
 c. Consequent: Alice will finish her thesis.

Claim A

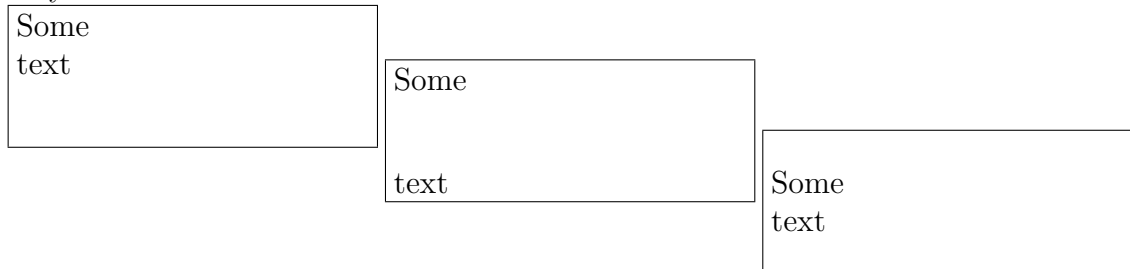
In those circumstances where the first sentence (**the antecedent**) is true, the second sentence (**the consequent**) is true.

So the first two lines of the truth table make perfect sense. Claim A is safe when both sentences are true, and it is clearly false when the antecedent is true and the consequent is false.

But what about when the first sentence is false? Here's what we're going to argue: if Alice doesn't win a fellowship, claim A is safe whether she finishes her thesis or not. Claim A only requires that IF she wins the fellowship, thesis-finishing follows. So if she didn't, the claim is still compatible with the facts ("true"), according to the truth table.

One case to consider: Suppose she doesn't win a fellowship. Does asserting (4a) commit us to the claim that she doesn't finish her thesis? [No. Suppose a millionaire donor gives her enough money to stop working and finish that thesis. (4a) doesn't rule that out. It just says that a fellowship would give her enough money to finish. It doesn't preclude other routes.]

Question: How well does this truth table accord with our intuitions about conditional sentences in English in general (*if ... then ...*) ? Answer: Not very.



	Ant.	Cons.	Conditional	Truth Value
(5)	(a) T	T	If 1960 was divisible by 5, then 1960 was a leap year.	T
	(b) F	T	If Al Gore won the election of 2000, George Bush won the election of 2004.	T
	(c) T	F	If George Bush won the election of 2004, Al Gore won the election of 2000.	F

(a) just seems false. (b) is weird not clear what kind of communicative act is being performed. (c) can be true as an instance of the “If X, I’ll eat my hat” construction.

3 Relations among Sentences

3.1 Contraries

Two sentences are contraries if they can’t both be true at the same time.

- (6) a. John is tall.
b. John is short.
- (7) a. John is happy.
b. John is sad.
- (8) a. This is a triangle.
b. This is a square.
- (9) a. Fido is a dog.
b. Fido is a country in Eastern Europe.

3.2 Contradictories

Two sentences are contradictories if they are contraries and can’t both be false.

- (10) a. John is a fool.

b. John is not a fool.

- (11) a. Some dog barked.
b. No dog barked.

Not contradictories, only contraries.

- (12) a. John is tall.
b. John is short.

What about?

- (13) a. John is very tall.
b. John is not very tall.

3.3 Entailment

1. A sentence A entails a sentence B if whenever A is true, B has to be true.
2. A sequence of sentences A_1, A_2, A_n entail a sentence B if whenever A_1, A_2, A_n are true, B has to be true.

Thanks to Gennaro Chierchia and Sally McConnell-Ginet for the following examples.

The first two sentences entail the second.

- (14) a. This is yellow.
b. This a fountain pen.
c. This is a yellow fountain pen.

How about these?

- (15) a. This is big.
b. This a sperm whale.
c. This is a big sperm whale.
- (16) a. Lee kissed Kim passionately.
b. Lee kissed Kim.

- c. Kim was kissed by Lee.
Kim was kissed.
Lee touched Kim with her lips.

Also consider this: Does Lee mouthing a kiss at Kim from 15 feet away count as kissing him?

- (17) a. Jane ate oatmeal for breakfast today.
b. Jane ate breakfast today.
- (18) a. Juan is not aware that Mindy is pregnant.
b. Mindy is pregnant.
- (19) a. Juan does not believe that Mindy is pregnant.
b. Mindy is pregnant.
- (20) a. Juan believes that Mindy is pregnant.
b. Mindy is pregnant.
- (21) a. Every second-year student who knows Latin will get credit for it.
b. If Juan is a second-year student and knows Latin, he will get credit for it.
- (22) a. If Alice wins a fellowship, she can finish her thesis.
b. If Alice doesn't win a fellowship, she can't finish her thesis.
- (23) a. If Alice finished her thesis, then she won a fellowship.
b. If Alice didn't win a fellowship, then she didn't finish her thesis.
- (24) a. Maria and Alberto are married.
b. Maria and Alberto are married to each other.
- (25) a. Only Amy knows the answer.
b. Amy knows the answer.
- (26) a. Amy knows the answer.
b. Only Amy knows the answer.
- (27) a. Mary is a Italian violinist.
b. Some Italian is a violinist.

- c. Some violinist is Italian.
- (28) a. Some student will not go to the party.
b. Not every student will go to the party.
- (29) a. Allegedly, John is a good player.
b. John is a good player.
- (30) a. Oscar and Jenny are rich.
b. Oscar is rich.
c. Jenny is rich.
- (31) a. Oscar and Jenny are middle-aged
b. Oscar is middle-aged.
c. Jenny is middle-aged.

3.4 Logical equivalence: The logical importance of and

- (32) a. Mary is an Italian violist.
b. Mary is a violinist.
c. Mary is Italian.
d. Mary is a violinist and Mary is Italian.
- (33) a. This is a yellow fountain pen.
b. This is yellow and this is a fountain pen.
- (34) a. John and Mary ate.
b. John ate.
c. Mary ate.
d. John ate and Mary ate.
e. John or Mary ate.
f. John ate or Mary ate.
- (35) a. John and Mary ate
b. $p = \text{John ate.}$
c. $q = \text{Mary ate.}$
d. $\llbracket \text{John and Mary ate.} \rrbracket = p \ \& \ q.$
e. $\text{eat}(\text{John}) \ \& \ \text{eat}(\text{Mary})$

(e) captures how the individual words contribute to the meaning.

4 Truth tables and trees: Is “ \sim ” working right?

If p and $\sim p$ are contradictories, they should never **both** be true. In other words:

$$p \ \& \ \sim p$$

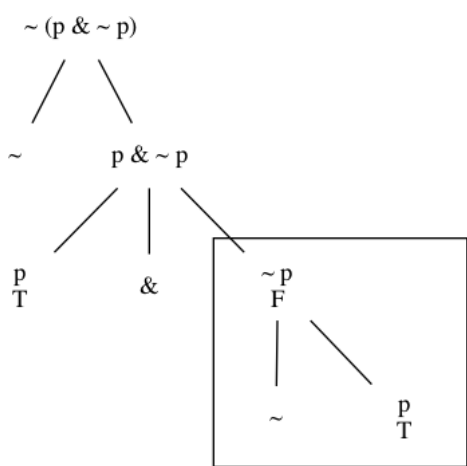
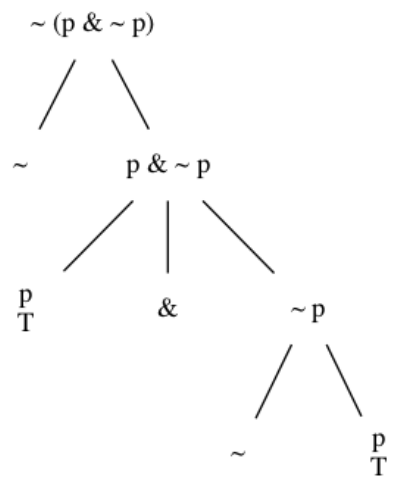
should always be false, whether p is T or F.

And if **that's** right, then

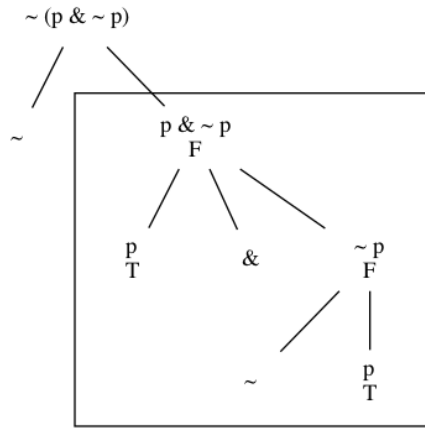
$$\sim (p \ \& \ \sim p)$$

should always be true, whether p is T or F.

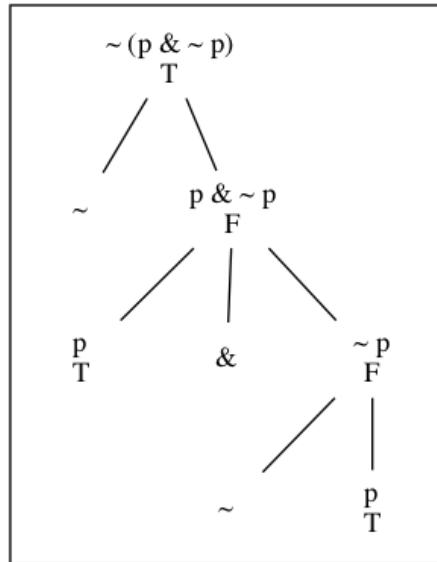
Does this turn out to be true?



p	$\sim p$
T	F
F	T



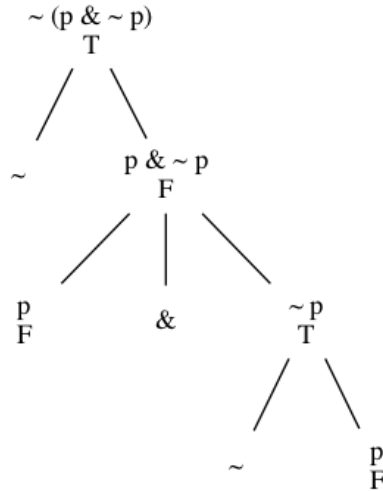
p	$\sim p$	$p \& \sim p$
T	T	T
T	F	F
F	T	F
F	F	F



$p \& \sim p$	$\sim(p \& \sim p)$
T	F
F	T

So far, so good.

Then, following the same method, we try making p false, and we get:



And we're done. No matter what the truth-value of p , $\sim(p \& \sim p)$ always turns out true. So if our goal is to design a system in which this always turns out true, we've done it.

What do I mean by "a system" here? Well, the system is statement logic. The relevant part of statement logic is the rules for assigning truth values to sentences containing $\&$ and \sim .

Terminological point: We call a sentence which **always** turns out to be true, no matter what the truth-values of the statements in it, a **valid statement**. Validity is a general concept used in all logical systems. Valid sentences in statement logic are called **tautologies**. We just **proved** $\sim(p \& \sim p)$ is a tautology.

The following table is called a **truth table**. It summarizes what we did in the two trees above; each row corresponds to one choice of a truth value for p . The last column gives the truth-value for $\sim(p \& \sim p)$ when p has the given truth value:

p	$\sim p$	$p \& \sim p$	$\sim(p \& \sim p)$
T	F	F	T
F	T	F	T

The truth table above tells us $\sim(p \& \sim p)$ is valid because the last column consists entirely of T's. Now look at the second to last column $p \& \sim p$. That column is filled entirely with F's. What that tells us is that $p \& \sim p$ always turns out to be false. Such a statement is called a **contradiction**. So we

found our tautology by negating a contradiction. That always works. Find yourself a contradiction and negate it, and you've got a tautology. It works to go the other way too. Negate a tautology and you've got a contradiction. So what is $\sim\sim(p \ \& \ \sim p)$?

We can use a truth table to check the validity of any statement. (call it ϕ and let it consist of atomic statements p, q, \dots), The idea is to run through all possible ways of assigning truth values to p, q, \dots , and if the last column, which tells us the truth values for ϕ , turns out to consist entirely of trues, then ϕ is a tautology.

Here's another example:

p	q	$p \rightarrow q$	$q \rightarrow (p \rightarrow q)$
T	T	T	?
T	F	F	?
F	T	T	?
F	F	T	?

Try to finish this table without looking at the answer on the next page. Note that the first three columns, filled in, are just the truth table for \rightarrow that we used above. To compute the last column, we need to use that truth table, because the main connective of $q \rightarrow (p \rightarrow q)$ is \rightarrow .

To fill in the last cell in the first row, look only at truth values in the first row. It tells us that q is true and $p \rightarrow q$ is true. So we have:

$$\begin{array}{ccc} q & \rightarrow & (p \rightarrow q) \\ \text{T} & & \text{T} \end{array}$$

The truth table for \rightarrow tells us that when the left hand side and the right hand side are both true, the entire statement is true. So we enter that as the truth value in the last column:

p	q	$p \rightarrow q$	$q \rightarrow (p \rightarrow q)$
T	T	T	T
T	F	F	?
F	T	T	?
F	F	T	?

Let's do the second row. We have.

$$\begin{array}{ccc} q & \rightarrow & (p \rightarrow q) \\ \mathbf{F} & & \mathbf{F} \end{array}$$

The truth table for \rightarrow tells us that when both the left hand side and the righthand side are false, the entire statement is true (this case is in boldface in the small truth table below). So we enter that in the last column.

p	q	$p \rightarrow q$	$q \rightarrow (p \rightarrow q)$
T	T	T	T
T	F	F	T
F	T	T	?
F	F	T	?

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

You should now be able to finish the truth table on your own. The answer is on the next page.

p	q	$p \rightarrow q$	$q \rightarrow (p \rightarrow q)$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

This too is a tautology. Notice that this time the truth table has four rows because there are four ways of assigning truth values to two distinct statements. How many rows would the truth table for $p \vee q \vee r$ have?

Now try this one all on your own. When you're done, ask your self if the sentence in the last column is a tautology, a contradiction, or neither.

p	$\sim p$	$p \vee \sim p$
T	?	?
F	?	?

Now do this one

p	q	$q \rightarrow p$	$(q \rightarrow p) \rightarrow q$
T	T		
T	F		
F	T		
F	F		

5 Contradictories revisited

Two things we showed in the last section were

1. p and $\sim p$ are contraries. We showed this when we showed that $p \& \sim p$ is a contradiction. p and $\sim p$ can't both be true.
2. p and $\sim p$ are contradictories. We showed this when we showed that $p \vee \sim p$ is a tautology, That is, either p is true or $\sim p$ is true.
3. The two laws we just proved about the relationship of p and $\sim p$ are important enough to deserve names.

The Law of Contradiction $\sim (p \& \sim p)$
 The Excluded Middle $p \vee \sim p$

6 Summary

1. We have truth tables defining a number of connectives: *not* (\sim), *and* ($\&$), *or* (\vee), and *implies* (\rightarrow)
2. Taken collectively these define **statement logic**.
3. A tautology is a statement whose truth table always turns out true.
4. A contradiction is a statement whose truth always turns out false.
5. In statement logic, **by definition**, $\sim p$ is the contradictory of p . We have proved this using truth tables by showing p and $\sim p$ can't both be true (the Law of Contradiction), and either p or $\sim p$ is true (the Excluded Middle).

7 Statement Logic Classification of sentences

7.1 Logical Equivalence

- (36) a. $p \rightarrow q$
 b. $\sim p \vee q$
 c.

	p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
(a)	T	T	F	T	T
(b)	T	F	F	F	F
(c)	F	T	T	T	T
(d)	F	F	T	T	T

- d. $(\sim p \vee q) \iff (p \rightarrow q)$

7.2 Logical truths (Tautologies)

The statement $p \rightarrow (q \rightarrow p)$, parenthesized THIS way, is always true. Such statements are called **tautologies**:

$p \rightarrow q$ rule

Example using only $p \rightarrow q$ rule

	p	q	$p \rightarrow q$
(a)	T	T	T
(b)	T	F	F
(c)	F	T	T
(d)	F	F	T

	p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
(a)	T	T	T	T
(b)	T	F	T	T
(c)	F	T	F	T
(d)	F	F	T	T

There are also English sentences that can't help but be true.

- (37) a. It's raining or it's not raining.
 b. Every Italian violinist is Italian.
 c. Every Italian violinist is a violinist.
 d. This yellow pencil is yellow.

Logicians call example (a) a **tautology** because its truth can be shown in a truth table. That's because it has to be true in virtue of the meaning of *and* and *not*, which are connectives in statement logic. The other examples are not like that. The reasons they have to be true is more complicated. Nevertheless they are called **logical truths** because they can be proved in a kind of logic called **predicate logic**, which we'll look at later. In the meantime, note that the syntactic form of these sentences has something to do with why they have to be true, but that's not the whole story. Example (37c) above has the same syntactic form as

- (38) Every fake gun is a gun.

but (38) is false. Maybe a fake gun is gunlike or gun-ish or similar to a gun, but it's not actually a gun. If it were, it wouldn't be fake.

8 Contingent sentences

Sentences like $(p \rightarrow q) \rightarrow p$, parenthesized THIS way, are called contingent sentences. They are sometimes true and sometimes false.

	p	q	$p \rightarrow q$	$(p \rightarrow q) \rightarrow p$
(a)	T	T	T	T
(b)	T	F	F	T
(c)	F	T	T	F
(d)	F	F	T	F

Another example:

	p	q	$p \rightarrow q$	$p \& (p \rightarrow q)$	$\sim q$	$(p \& (p \rightarrow q)) \rightarrow \sim q$
(a)	T	T	T	T	F	F
(b)	T	F	F	F	T	T
(c)	F	T	T	F	F	T
(d)	F	F	T	F	T	T

Most English sentences are contingent.

- (39) a. Every Italian violinist is temperamental
 b. It's raining.
 c. This yellow pencil is mine.

8.1 Contradictions

Some sentences have truth tables that always make them false. Such sentences are called **contradictions**, because they can't help but be false:

	p	$\sim p$	$p \& \sim p$
(a)	T	F	F
(b)	F	T	F

Another example:

	p	q	$p \rightarrow q$	$\sim (p \rightarrow q)$	$q \& \sim (p \rightarrow q)$
(a)	T	T	T	F	F
(b)	T	F	F	T	F
(c)	F	T	T	F	F
(d)	F	F	T	F	F

There are also English sentences that can't help but be false. These too are called contradictions.

- (40) a. Some Italian violinist is not a violinist.
 b. It's raining and it's not raining.
 c. This yellow pencil is not yellow.
 d. That triangle has four sides.

Notice that some of these contradictions are just the negations of logical truths [(a) and (c)]. Example (b) is the conjunction of contradictories. And example (d) asserts contrary properties of a single entity. So it's easy to generate contradictions of your own, and impress your friends.

8.2 Logical entailment

The statement $p \& q$ entails the statement $p \vee q$. If the first is true, then obviously the second has to be true. We can show this by looking at the truth table for both:

	p	q	$p \& q$	$p \vee q$
(a)	T	T	T	T
(b)	T	F	F	T
(c)	F	T	F	T
(d)	F	F	F	F

Checking each row: Whenever $p \& q$ is true (boldfaced cell in row a), $p \vee q$ is true. We write this as follows:

$$p \& q \Rightarrow p \vee q$$

Nevertheless, when it comes to English sentences using the word *or*, we need to account for the fact that *or* often communicates exclusivity (either one or the other, but not both). We will call this an implicature. Here are some examples of how the claim works.

- (41) a. Either he rented either a mid-size car or he rented an economy car. [$p \vee q$ translation claims: If in fact he rented both, this is still literally true; but by implicature, he can't have rented both.]

- b. Either there's no bathroom in this house or it's on the second floor. [In fact both statement can't be simultaneously true, but that's not due to the meaning of *or*]
- c. You can have either the white one or the red one. [intended meaning: but not both, an implicature]

Exploiting the semantics/pragmatics division of labor. boundary: We always translate 'A or B' as $\llbracket A \rrbracket \vee \llbracket B \rrbracket$ The *exclusive-or* interpretation is an implicature that holds only in certain contexts. In those contexts C:

$$\llbracket A \rrbracket \vee \llbracket B \rrbracket \Rightarrow_C \sim (\llbracket A \rrbracket \& \llbracket B \rrbracket)$$

If add the information that A and B aren't both true to the meaning of *or*, you get an exclusive or reading.

This is a **quantity implicature**. Why?

The idea is the following. If we say $p \vee q$, then we choose not to say $p \& q$, which is more informative. (The fact that $p \& q$ is more informative than $p \vee q$ is pretty intuitive, another way of saying this is that $p \& q$ entails $p \vee q$ or $p \& q \Rightarrow p \vee q$). But if both p and q actually **are** true, and we choose to say only $p \vee q$, that would violate the Maxim of Quantity (be as informative as possible). So saying $p \vee q$ implicates that we have some reason for not saying $p \& q$, most likely that p and q are not both true. It is a defeasible implicature. If the context overrules the implicature, then $p \& q$ might be possible. For example,

(42) Students must have a pencil or a pen.

In this case context excludes the implicature *students are not allowed to have both a pencil and a pen*. This is natural: a rule that forbids having both would be *more* restrictive. But notice this peculiar reading is exactly what we would get if *or* meant *exclusive or*.

8.3 Summary

We would like our logical translations to capture the following kinds of semantic facts:

1. Explain entailment patterns.

2. Explain contradictions and sentences pairs that are contradictory or contraries.
3. Explain tautologies.
4. Account for compositionality: How the individual words and the structural meaning contribute to the meaning as a whole.

9 Doing Semantics: Applying Logical Entailment

We now have ways of writing down and investigating some of our key ideas. In this section we demonstrate this by arguing that if q is a contrary of p , then

$$(43) \quad q \text{ entails } \sim p$$

This is an important idea. It gives us lots of simple semantic facts. Once we know that *short* and *tall* are contraries, we know that *John is not short* entails *John is not tall*, and similarly for all other contraries.

We start with the fact that q is some contrary of p , using the definition of a contrary of p :

$$\sim (p \ \& \ q) \tag{1}$$

We want to investigate situations in which (1) is true and q is also true. That is we're interested in situations in which

$$q \ \& \ \sim (p \ \& \ q) \tag{2}$$

is true.

What we want to show is:

$$(q \ \& \ \sim (p \ \& \ q)) \Rightarrow \sim p$$

So we investigate the truth tables for $(q \ \& \ \sim (p \ \& \ q))$ and $\sim p$ to see if the second is sentence is true in every row where the first sentence is. How many rows do we need? Well, we only have two statement letters, so 4.

p	q	$p \& q$	$\sim (p \& q)$	$(q \& \sim (p \& q))$	$\sim p$
T	T	?	?	?	?
T	F	?	?	?	?
F	T	?	?	?	?
F	F	?	?	?	?

So we want to show that whenever there is a T in the second to last column there is also a T in the last column.

We'll do the first column together.

p	q	$p \& q$	$\sim (p \& q)$	$(q \& \sim (p \& q))$	$\sim p$
T	T	T	?	?	?
T	F	F	?	?	?
F	T	F	?	?	?
F	F	F	?	?	?

Now finish this.

10 Ambiguity

- (44)
- a. You should have seen that bull we got from the pope.
 - b. Competent women and men hold all the good jobs in the firm.
 - c. Mary claims that John saw her duck.
 - d. John and Mary are married.

Ambiguous sentences have more than one meaning. That means if we translating into an unambiguous logical language, they should get more than one translation. This is one of the ways in which the logical analysis is a help, it gives us a tool to spell out what the distinct readings are and maybe why they exist.

Consider (44d), a case we are going to want to call ambiguous, but which not seem so obviously a case of ambiguity at first.

Pattern 1 (very general)

- (45)
- a. A is Adj.
 - b. B is Adj.

- c. A and B are Adj.
- d. happy, tall, incompetent, married, annoying,...

Pattern 2 (symmetric predicates)

- (46) a. A is Adj Prep B.
- b. A and B are Adj.
- c. similar to, married to, adjacent to, different from

Unambiguous

- (47) a. John and Mary are annoying.
- b. John and Mary are similar.

Question: Why is (b) unambiguous? How does *similar to* differ from *married to*?

Revised view?

- (48) a. Maria and Alberto are married.
- b. Maria and Alberto are married to each other.