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1 Exercise 1

Which of the following cannot be translated like the others?

a. Oliver and Richard are Roundheads.

b. Oliver and Richard are relatives.

c. Oliver and Richard like to drink

Consider the following attempted translations. (B) is the outlier:

A. \( p = \) Oliver is a Roundhead.
\( q = \) Richard is a Roundhead.
\( p \& q \)

B. \( p = \) Oliver is a relative.
\( q = \) Richard is a relative.
\( p \& q \)

C. \( p = \) Oliver likes to drink.
\( q = \) Richard likes to drink.
\( p \& q \)

Note that B. has been done so as to be parallel to A. and C., but the translation \( p \& q \) in B. is not a paraphrase of sentence (b).
2 Exercise 2 (a)-(e)

a. If this is summer it’s damned cold.

\[ p = \text{This is summer.} \]
\[ q = \text{It’s damned cold.} \]
\[ p \rightarrow q \]

b. Lemons look good but taste sour.

\[ p = \text{Lemons look good.} \]
\[ q = \text{Lemons taste sour.} \]
\[ p \land q \]

c. You can if you want to.

\[ p = \text{You can [do X].} \]
\[ q = \text{You want to [do X].} \]
\[ q \rightarrow p \]
Wrong: \[ p \rightarrow q \]

d. He will come today or tomorrow but not later.

\[ p = \text{He will come today.} \]
\[ q = \text{He will come tomorrow.} \]
\[ r = \text{He will come later.} \]
\[ (p \lor q) \land \neg r \]
Wrong: \[ r = \text{He will not come later.} \]
Wrong: \[ r = \text{not later} \]

e. If neither God nor the Devil exists, it is difficult to be religious.

\[ p = \text{God exists.} \]
\[ q = \text{The Devil exists.} \]
\[ r = \text{It is difficult to be religious.} \]
Two equivalent correct answers
\[ \neg (p \lor q) \rightarrow r \]
\[ (\neg p \land \neg q) \rightarrow r \]
f. Throw the cat out or I will leave.

\[
\begin{align*}
p & = (\text{You}) \text{ throw the cat out.} \\
q & = \text{ I will leave.} \\
\text{Two equivalent correct answers} \\
p \lor q \\
\sim p \rightarrow q
\end{align*}
\]

3 Exercise 3 (d), (f)

In the following, we assume \( p, q \) are true, and \( r \) is false:

a. \( \sim p \)

\[
\begin{array}{c|c|c}
p & \sim p \\
T & F
\end{array}
\]

b. \( \sim (p \land r) \)

\[
\begin{array}{c|c|c|c}
p & r & (p \land r) & \sim (p \land r) \\
T & F & F & T
\end{array}
\]

c. \( \sim (p \lor q) \)

\[
\begin{array}{c|c|c|c}
p & q & p \lor q & \sim (p \lor q) \\
T & T & T & F
\end{array}
\]

d. \( p \lor (q \land r) \)

\[
\begin{array}{c|c|c|c|c}
p & q & r & (q \land r) & p \lor (q \land r) \\
T & T & F & F & T
\end{array}
\]

e. \( r \rightarrow ((q \land r) \lor (p \lor q)) \)

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
p & q & r & (q \land r) & (p \lor q) & (q \land r) \lor (p \lor q) & r \rightarrow ((q \land r) \lor (p \lor q)) \\
T & T & F & F & T & T & T
\end{array}
\]

ctd. next page
4 Exercise 4

Which of the following are tautologies?

a. $\sim(p \& \sim p)$

<table>
<thead>
<tr>
<th>p</th>
<th>$\sim p$</th>
<th>$p &amp; \sim p$</th>
<th>$\sim(p &amp; \sim p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
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<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Tautology

b. $(p \lor q) \rightarrow p$

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>$(p \lor q)$</th>
<th>$(p \lor q) \rightarrow p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
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<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Contingent

c. $\sim(p \& q)$

$\equiv (\sim p \lor \sim q)$

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>$(p &amp; q)$</th>
<th>$\sim p$</th>
<th>$\sim q$</th>
<th>$(\sim p \lor \sim q)$</th>
<th>$\sim(p &amp; q)$</th>
<th>$\sim(p &amp; q) \equiv (\sim p \lor \sim q)$</th>
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</thead>
<tbody>
<tr>
<td>T</td>
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</tbody>
</table>

Tautology
More compactly (same number of columns):

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>q</td>
<td>(p &amp; q)</td>
<td>~p</td>
<td>~q</td>
<td>(b) &amp; (c)</td>
<td>~p &amp; ~q</td>
<td></td>
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<tr>
<td>T</td>
<td>T</td>
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</tbody>
</table>

\[ \sim (p \land q) \equiv (\sim p \lor \sim q) \]

\[ \sim p \land q \]

\[ (p \land q) \]

\[ \sim p \]

\[ \sim q \]