Modality Assignment

Jean Mark Gawron

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1 Intro

Thanks to Kate Kearns for many of these examples.

2 Basic

Write the truth definitions using possible worlds for these sentences. In each case identify the kind of modality and if there is more than one possibility, give truth definitions for both kinds.

(1) a. Necessarily, a bachelor is unmarried.

Logical	'Necessarily, a bachelor is unmarried' is true iff $\forall w$['a bachelor is unmarried' is true in w]

b. A Romanian could have invented the hypodermic needle.

Epistemic	'A Romanian could have invented the hypodermic needle' is true iff $\exists w_e$['A Romanian invented the hypodermic needle" is true in w_e]
Logical	'A Romanian could have invented the hypodermic needle' is true iff $\exists w$['A Romanian invented the hypodermic needle" is true in w]

c. If wishes were horses then beggars would ride.

Logical	'If wishes were horses then beggars would ride.' is true iff $\forall w$['Wishes are horses' is true in $w, \rightarrow$ 'Beggars ride' is true in w]

d. It is sure to rain tonight.

Epistemic	'It is sure to rain tonight.' is true iff $\forall w_e$['It will rain tonight' is true in w_e]

e. Students' laptops may be brought to the dance.

Deontic	'Students' laptops may be brought to the dance. is true iff $\exists w_{po}$['Students' laptops are brought to the dance.' is true in w_{po}]
Epistemic	'Students' laptops may be brought to the dance. is true iff $\exists w_e$['Students' laptops are brought to the dance.' is true in w_e]

f. Left-turning traffic must yield.

Deontic	'Left-turning traffic must yieldLeft-turning traffic must yield'. is true iff $\forall w_{po}$['Left-turning traffic yields' is true in w_{po}]
Epistemic	'Left-turning traffic must yield. is true iff $\forall w_e$['Left-turning traffic yields.' is true in w_e]

g. John may go. (= it is permissible for John to go.) Deontic
(= it is possible that John will go.) Epistemic

Deontic	'John may go.' is true iff $\exists w_{po}$['John will go' is true in w_{po}]
Epistemic	'John may go.' is true iff $\exists w_e$['John will go.' is true in w_e]

h. John may not go. (= it is not permissible for John to go.)
(= it is possible that John will not go.)

Deontic	'John may go.' is true iff $\sim \exists w_{po}$['John will go' is true in w_{po}]
Epistemic	'John may go.' is true iff $\exists w_e$['John will go.' is not true in w_e]

i. It is permissible for John to go.

Deontic	'It is permissible for John to go.' is true iff $\exists w_{po}$['John will go' is true in w_{po}]

j. It is not permissible for John to go. (= It is forbidden for John to go.)

Deontic	'It is not permissible for John to go.' is true iff $\sim \exists w_{po}$['John will go' is true in w_{po}] is true iff $\forall w_{po} \sim$ ['John will go' is true in w_{po}] (Since $\sim \exists w_{po} \equiv \forall w_{po} \sim$) is true iff $\forall w_{po}$['John will not go' is true in w_{po}] is true iff $\forall w_{po}$['John will go' is false in w_{po}]

k. It is permissible for John not to go.

Deontic	'It is permissible for John not to go.' is true iff $\exists w_{po}$['John will go' is false in w_{po}] is true iff $\exists w_{po} \sim$ ['John will go' is true in w_{po}]

l. It is not permissible for John not to go. (= It is not the case that John may not go.)
(= John must go.)

Deontic	'It is not permissible for John not to go.' is true iff $\sim \exists w_{po}$['John will go' is false in w_{po}] is true iff $\sim \exists w_{po} \sim$ ['John will go' is true in w_{po}] is true iff $\forall w_{po}$['John will go' is true in w_{po}] (Since $\sim \exists w_{po} \sim \equiv \forall w_{po}$)

3 Would

Does *would* have the same kind of meaning in the following? What are the meanings? Is one modal? Are both modal?

(2) a. If the weather had been better the picnic would/could have been a success.
b. We knew Jack would/could arrive on time. [Keep in mind the presupposition properties of *know*].

would The first *would* is a modal. The second is a back-shifted future. It expresses the fact that we at a past time know something that was going to happen in the future (relative to that time).

could The first *could* is a modal. The second is ambiguous between an epistemic modal, and a past tense for ability *can*. More on ability can below: Basically (b) with that reading expresses the fact that we at a past time know that Jack had the ability to arrive on time.

(3) a. If she had studied harder she could have passed the semantics exam.
b. She could bench press close to 300 pounds.

First sentence The first *could* is a modal, logical modality. In the counterfactual possible worlds where she studies harder, she passes.

Second sentence This *could* is the past tense of ability *can*. Note that *ability* can is different from an epistemic modal. It's perfectly consistent to say *She might arrive on time and she might not*. That's an epistemic possibility statement. Based on what you know, it's possible that she'll arrive on time, but it's also possible she won't. But it's not consistent to say *She could bench press close to 300 pounds and she couldn't bench press close to 300 pounds*. Either she could or she couldn't. Note there's no uncertainty being expressed, as there is in the epistemic claim: This is a claim about ability in the actual world.

4 Necessary truth

Identify the ones that are necessary truths. If a sentence is not necessarily true, describe a possible world in which it is not true.

People got confused about this question because they had just answered a question about epistemic and logical modality. The term "necessary truth" is used by philosophers of language to describe **non modal** sentences that are true in a special way. Here's how Bill Talbott of the U. of Washington defines it for his Epistemology course.

> A necessary truth is a proposition that could not possibly have been false. This can be expressed by saying that a necessary truth is a proposition that is true in every possible world. An example of a truth that many philosophers take to be necessary in this sense is: 2+2 = 4.

Notice that the definition of necessary truth makes reference to possibility: *could not possibly have been false*. This does not mean that necessary truths are modal sentences. "Two plus two is four" is not a modal sentence. But we can ask about any sentence: What worlds is it true in? And the ones that turn out to be true in all worlds are the necessary sentences.

The opposite of a necessary truth is a contingent truth, something that might have turned out differently, something that is true in some possible worlds and false in others. When we talk about what's possible in the context of necessary truth, we mean all possible worlds, not just those consistent with what we know to be the facts, so counterfactual possibilities are included.

Many of you seemed to think that *necessary* meant *epistemically necessary*. In other words, if we know something to be true, then it's a necessary truth. That's wrong, as the first example shows. There are things we know to be true which might have turned out differently. Given that background, here are the answers to the problem below.

George Bush won the 2000 election. Clearly contingent, not necessary. Bush very narrowly won that election. If a few **hundred** votes in Florida had been cast differently, or maybe even counted differently, the outcome of the entire election would have been altered.

All men are mammals. Probably necessary. Are there possible worlds in which it turns out that men are robots placed on earth by an alien race millenia ago (while women are mammals)? You decide.

A solid body occupies space. Necessary in virtue of the meaning of the words.

A dog is four-legged. Many of you were quite sure this is not necessary, because there are three-legged dogs. But before we get to that, is this sentence true, and if so, how is it true, despite the fact that there are three-legged dogs in this very world? I think it is true, and it is true in an interesting way. The fact is, this is a **generic** sentence. It doesn't mean the same as *Every dog is four-legged* (which is false). It means something more like *As a rule, dogs are four-legged* or *A typical dog* (whatever that is) *is four-legged*. So, leaving aside how we could ever give precise truth-conditions for such a claim, it **is** true. We say the sentence is true on the **generic** reading. And the way to answer this is to ask whether the generic claim is necessarily true. Are there possible worlds in which the typical dog is three-legged or five-legged or no-legged?

A dog is a canine. It is tempting to say: However you answered the *All men are mammals* questions, that's how you should answer this one. But this one is a generic too, so the more interesting comparison is to *Dogs are four-legged*. Are there possible worlds in which the typical dog is not a canine but an ursine (a kind of bear)? What would such a world be like? What would pit bulls be like? Or does it make sense to say that while both generics are true, *A dog is a canine* is a necessary truth, while *A dog is four-legged* is not.

Orange is the color of oranges. Many of you agreed with me that oranges might easily have been blue or striped and still taste every bit as sweet. One of you pointed out the example of blood oranges, which is a good one. Contingent.

5 Meaning and possible worlds

In the homework assignment previously we identified the meaning of a sentence (or at least its sense, or intension) with the set of worlds in which it is true.

What can you say about the meanings of the following sentences, *assuming this theory*?

(4) a. Either God exists or God does not exist,
b. Every rose is a rose.

c. The sum of two and two is four.

Answer: All of these sentences are necessarily true. I want to thank those students who provided stimulating theological discussion about example (4a), but the truth of the matter is that it's a logical truth, not because of what it's about but because of its form. Any sentence of the form:

$$p \vee \sim p$$

is a tautology, true whatever the truth value of p. And logical truths are an important kind of necessary truth, true in all possible worlds. Similarly, (4b) is another kind of logical truth, true in virtue of its form. Any sentence of the form

$$\forall x[\mathrm{P}(x) \rightarrow \mathrm{P}(x)]$$

is a truth of logic, whatever P is, and therefore true in all possible worlds. And (4b) has this form. Finally (4c) is a truth of arithmetic, another kind of necessary truth. So all three sentences are true in all possible worlds. And that's the problem with using possible worlds to ground your theory of meaning. If the set of possible worlds a sentence is true in its **sense**, or **meaning**, then we can't distinguish the meanings of those three sentences.

6 Modality and truth

Give the truth definitions for the sentences below, assuming the possible worlds theory of modality. Discuss the adequacy of this account.

(5) a. If squares were circles then cubes would be spheres.

Logical 'If squares were circles then cubes would be spheres.' is true iff $\forall w$['Squares are circles' is true in $w \rightarrow$ 'cubes are spheres' is true in w]

b. If squares were circles then cubes would be cylinders.

Logical 'If squares were circles then cubes would be cylinders.' is true iff $\forall w$['Squares are circles' is true in $w \rightarrow$ 'cubes are cylinders' is true in w]

c. If Bizet and Verdi were compatriots, Verdi would be French.

Logical ‘If Bizet and Verdi were compatriots then Verdi would be French.’ is true iff $\forall w$[‘Bizet and Verdi are compatriots’ is true in $w \rightarrow$ ‘Verdi is French’ is true in w]

d. If Bizet and Verdi were compatriots, Bizet would be Italian.

Logical ‘If Bizet and Verdi were compatriots then Verdi would be French.’ is true iff $\forall w$[‘Bizet and Verdi are compatriots’ is true in $w \rightarrow$ ‘Bizet is Italian’ is true in w]

Discussion : First of all, each truth definition is a fairly straightforward extension of the treatment of conditional sentences outlined in the book. A conditional is true if all worlds in which the antecedent clause is true, the consequent clause is true. For (a), we might also write:

$$\forall\, w \in [\![\text{squares are circles}]\!]\, [\, w \in [\![\text{cubes are spheres}]\!]\,]$$

These two pairs of examples are about the same thing, a problem with our treatment of counterfactuals. Take (c) and (d): according to our treatment, it is impossible for both sentences to be true. If it turns out that (c) is true, then in all the compatriot worlds, both Bizet and Verdi are French. If it turns out (d) is true, then in all compatriot worlds, both Bizet and Verdi are Italian. Now this doesn't reflect any kind of linguistic reality. In fact, one can imagine (c) being true in one kind of context, and (d) being true in another. What kind of contexts are those, and how should we fix our account?