1 Part one: Entailments [35 points]

1.1. (a) Susan and Barb walked to the corner.
      (b) Susan walked to the cornner.
      (c) $a \Rightarrow b$
      (d) # Susan and Barb walked to the corner but Susan didn’t walk to the corner.

1.2. (a) Fred knitted a blue sweater.
      (b) Fred knitted a sweater.
      (c) $a \Rightarrow b$
      (d) # Fred knitted a blue sweater but he didn’t knit a sweater.

1.3. (a) Fred gave Lena a bicycle.
      (b) Lena received a bicycle from Fred.
      (c) $a \iff b$
      (d) # Fred gave Lena a bicycle but she didn’t receive a bicycle from him.

1.4. (a) John lost his wallet.
      (b) John lost his wallet in the tunnel.
      (c) $b \Rightarrow a$
(d) # John lost his wallet in the tunnel but he didn’t lose his wallet.

1.5. (a) \( b \Rightarrow a \)
(b) The water boiled.
(c) Susan boiled the water.
(d) # Susan boiled the water but it didn’t boil.

1.6. (a) No house with more than two bedrooms sold that month.
(b) No house with more than one bedroom sold that month.
(c) \( b \Rightarrow a \)
(d) # No house with more than one bedroom sold that month but a house with more than two bedrooms did sell.

1.7. (a) Apples were discovered in the pantry.
(b) John discovered apples in the pantry.
(c) \( b \Rightarrow a \)
(d) # John discovered apples in the pantry, but no apples were discovered in the pantry.
(e) (Alternative less natural cancellation attempt) # John discovered apples in the pantry, but apples weren’t discovered in the pantry.

1.8. (a) Sandy married Kim.
(b) Kim married Sandy.
(c) \( a \iff b \)
(d) # Sandy married Kim but Kim didn’t marry Sandy.

1.9. (a) Sandy liked Kim.
(b) Kim liked Sandy.
(c) Neither entails the other.
(d) Successful cancellation (Not a necessary part of your answer): Kim liked Sandy, but Sandy didn’t like Kim.

1.10. (a) Sandy and Kim exchanged phone numbers.
(b) Sandy gave Kim her phone number.
(c) \( a \Rightarrow b \)

(d) # Sandy and Kim exchanged phone numbers but Sandy didn’t give Kim her phone number.

2 Logic section of the practice exam [30 points]

Consider the truth table for \( \sim q \rightarrow (\sim p \& \sim q) \)

\[
\begin{array}{c|c|c|c|c|c}
 p & q & \sim p & \sim q & (\sim p \& \sim q) & \sim q \rightarrow (\sim p \& \sim q) \\
\hline
 T & T & F & F & F & T \\
 T & F & F & T & F & F \\
 F & T & T & F & T & T \\
 F & F & T & T & T & T \\
\end{array}
\]

Answer the questions below about the following expressions. [20 points]
truth tables  
Classification of logical relation (equivalent, tautology, contradiction, or non of the above)  

(a) \( q \rightarrow p \)

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Not Equivalent

(b) \(~ (p \& \sim q)\)

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<th>( \sim q )</th>
<th>( p &amp; \sim q )</th>
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Equivalent

(c) \((p \lor q) \rightarrow q\)

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Equivalent

(d) \((p \& q) \rightarrow (p \lor q)\)

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<th>( p \lor q )</th>
<th>((p &amp; q) \rightarrow (p \lor q))</th>
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Tautology  
All rows in final col. true

(e) \(~ (p \rightarrow q) \& \sim p\)

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<th>( q )</th>
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<th>( p \rightarrow q )</th>
<th>(~ (p \rightarrow q) )</th>
<th>( \sim (p \rightarrow q) &amp; \sim p)</th>
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Contradiction  
All rows in final col. false

2.1. Determine which of the above expressions is logically equivalent to

\(~ q \rightarrow (\sim p \& \sim q)\)
Prove your answer by showing truth tables for all of the above expressions. See truth tables above.

2.2. Point out any of these expressions that are tautologies or contradictions and explain why using the truth tables. See truth tables above.

2.3. What is the truth value of the expression below when \( p, q, \) and \( r \) are all true. [10 points]

\[(\sim p \& r) \rightarrow (r \rightarrow p)\]

\[(\sim p \& r) \rightarrow (r \rightarrow p)\]
\[(\sim T \& T) \rightarrow (T \rightarrow T)\]
\[(F \& T) \rightarrow (T \rightarrow T)\]
\[F \rightarrow (T \rightarrow T)\]
\[F \rightarrow T\]
\[T\]

3 Translation section of the practice exam

Translate the following sentences into predicate logic of the sort introduced in Allwood, Anderson, and Dahl, and further discussed in chapters 2 & 3 of Kearns. For any ambiguous sentences, give all the readings, and paraphrase them, saying which logical translation goes with which reading. Except where indicated otherwise, translate definite NPs and proper names using single letter constants. If you have an issue about how to translate a word, please discuss it and justify your decision, rather than just, say, ignoring the word and losing unnecessary points. If you feel that a word is being used inconsistently and you need to give it more than one translation (such as when you translate transitive and intransitive \textit{eat} as \textit{EAT} and \textit{EAT2}), please explain why.

3.1. The car struck both the tree and the lamppost.

\[\text{strike}(c, t) \& \text{strike}(c, l)\]
\[c = \text{the car, } t = \text{the tree, } l = \text{the lamppost}\]
3.2. Joseph sat on either the tree stump or the swing.

\[ \text{sit-on}(j, t) \lor \text{sit-on}(j, s) \]
\[ j = \text{Joseph}, \ t = \text{the tree stump}, \ s = \text{the swing} \]

3.3. A broken tooth was found.

\[ \exists x [\text{tooth}(x) \land \text{broken}(x) \land \exists y \text{find}(y, x)] \]

Alternatively a new relation \textit{found} for the intransitive passive predicate:

\[ \exists x [\text{tooth}(x) \land \text{broken}(x) \land \text{found}(x)] \]

What’s unacceptable is not noticing this was a passive and just using a one-place relation for the transitive verb \textit{find}:

\[ \exists x [\text{tooth}(x) \land \text{broken}(x) \land \text{find}(x)] \]

Or using the passive predicate \textit{found} as the name for the transitive relation

\[ \exists x [\text{tooth}(x) \land \text{broken}(x) \land \exists y \text{found}(y, x)] \]

3.4. No student attended the dance.

\[ \sim \exists x [\text{student}(x) \land \text{attend}(x, d)] \]
\[ d = \text{the dance} \]

3.5. John was close to the water fountain.

\[ \text{close-to}(j, w) \]
\[ w = \text{the water fountain} \]

3.6. Sincerity frightens John. (treat \textit{sincerity} as if it were a proper name).

\[ \text{frighten}(s, j) \]
\[ s = \text{sincerity}, \ j = \text{John} \]

3.7. Susan will regret every broken promise.

\[ \forall x [(\text{promise}(x) \land \text{broken2}(x)) \rightarrow \text{regret}(s, x)] \]
\[ s = \text{Susan} \]

Arguably, this is a different sense of \textit{broken} than in \textit{broken tooth}

Note the contextually restricted quantification.

\textit{every broken promise} here means every broken promise made by Susan
3.8. An adolescent confessed to the crime.
\[ \exists x \left[ \text{adolescent}(x) \land \text{confess-to}(x, c) \right] \]
\[ c = \text{the crime} \]

3.9. Bill will be promoted.
\[ \exists y \text{ promote}(y, b) \]

Another passive: *promote* is a transitive verb. Alternatively a new predicate for the intransitive passive predicate:
\[ \text{promoted}(b) \]

3.10. Picasso painted a disturbing picture.
\[ \exists x \left[ \text{picture}(x) \land \text{disturbing}(x) \land \text{paint}(p, x) \right] \]