1 Logical translations missing a connective

\[ \sim (p \sim q) \rightarrow q \quad \text{bad} \]
\[ (p \sim q) \quad \text{bad} \]
\[ (p \land q) \quad \text{bad} \]
\[ \sim q \quad \text{OK!} \]

2 No work shown on (3). Given \( p = \text{true}; q = \text{true}; r = \text{false} \), how did you compute the truth of \( p \lor (q \land r) \)? You must have first computed the truth of \( (q \land r) \). Show that. Minimally:

\[ t \lor (t \land f) \]
\[ t \lor f \]
\[ t \]

3 “neither god nor the devil exists” wrong (see below).
4 No work shown on (4). The tautology problem. Must show complete truth tables. (two rows on a., four rows on b, and c.)

5 p, q not spelled out in 2.

Wrong
Neither the Patriots nor the Packers were in the Superbowl
\sim (p \lor q)

Right
Neither the Patriots nor the Packers were in the Superbowl
\sim (p \lor q)

\[p = \text{The Patriots were in the Superbowl}\]
\[q = \text{The Packers were in the Superbowl}\]
Exercise 1

(1)  
   a. Oliver and Richard are roundheads.
   b. Oliver and Richard are relatives.
   c. Oliver and Richard like to drink to each other.

Paraphrasing the sentences in (1) as conjoined sentences $p$ and $q$, as in (2), works for (a) and (c), and fails in the case of (b):

(2)  
   a. $p$ = Oliver is a roundhead.
       $q$ = Richard is roundhead.
       $p \& q$

   b. $p$ = Oliver is a relative.
       $q$ = Richard is a relative.
       $p \& q$

   c. $p$ = Oliver likes to drink
       $q$ = Richard likes to drink.
       $p \& q$
## Exercise Two

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a.</strong> $p \rightarrow q$</td>
<td>$\Rightarrow$</td>
<td>If this is summer, it’s damned cold.</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>$\Rightarrow$</td>
<td>this is summer</td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td>$\Rightarrow$</td>
<td>it’s damned cold.</td>
<td></td>
</tr>
<tr>
<td><strong>b.</strong> $p \land q$</td>
<td>$\Rightarrow$</td>
<td>Lemons look good, but taste sour.</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>$\Rightarrow$</td>
<td>Lemons look good.</td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td>$\Rightarrow$</td>
<td>Lemons taste sour.</td>
<td></td>
</tr>
<tr>
<td><strong>c.</strong> $q \rightarrow p$</td>
<td>$\Rightarrow$</td>
<td>You can if you want to.</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>$\Rightarrow$</td>
<td>You can do x</td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td>$\Rightarrow$</td>
<td>You want to do x.</td>
<td></td>
</tr>
<tr>
<td><strong>d.</strong> $(p \lor q) \land \neg r$</td>
<td>$\Rightarrow$</td>
<td>He will come today or tomorrow but not later.</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>$\Rightarrow$</td>
<td>He will come today</td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td>$\Rightarrow$</td>
<td>He will come tomorrow</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>$\Rightarrow$</td>
<td>He will come later than tomorrow</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>$\neq$</td>
<td>He will not come later than tomorrow</td>
<td></td>
</tr>
</tbody>
</table>
e. \( \sim (p \lor q) \rightarrow r \) = If neither God nor the devil exists, it’s difficult to be religious.
\( \sim (p \land q) \rightarrow r \neq \) If neither God nor the devil exists, it’s difficult to be religious.

\( p \) = God exists.
\( q \) = The Devil exists
\( r \) = It’s difficult to be religious.

f. \( p \lor q \) = Throw the cat out or I will leave.
\( \sim p \rightarrow q \) = Throw the cat out or I will leave.
\( p \) = [You] throw the cat out.
\( q \) = I will leave.
Exercise 3

a. \( \neg p(t) \)

b. \( \neg \text{Conj}(f) \)

   \( p(t) \& r(f) \)

   \( \neg \text{Neg}(t) \)

   \( \text{Neg}(f) \)


c. \( \neg \text{Disj}(t) \)

   \( p(t) \lor r(f) \)

   \( \text{Neg}(f) \)
Exercise 3, ctd.

d. 

\[
\begin{align*}
\text{Disj}(t) & \quad \text{Conj}(f) \\
p(t) & \quad \lor \\
q(t) & \quad \land \\
r(f) & \quad \\
\end{align*}
\]

e. 

\[
\begin{align*}
\text{Impl}(t) & \quad \text{Disj}(t) \\
r(f) & \quad \\
\text{Conj}(f) & \quad \lor \\
q(t) & \quad \land \\
r(f) & \quad \\
p(t) & \quad \lor \\
q(t) & \\
\end{align*}
\]
Exercise 3, ctd.

\[
\begin{array}{c}
\text{Eq}(t) \\
\hline
\text{r}(f) \leftrightarrow \text{Conj}(f) \\
\hline
\text{p}(t) \& \text{r}(f)
\end{array}
\]
Exercise 4a: Tautologies

\( \sim (p \& \sim p) \)

<table>
<thead>
<tr>
<th></th>
<th>( \sim p )</th>
<th>( p &amp; \sim p )</th>
<th>( \sim (p &amp; \sim p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Is a tautology!
### Exercise 4b: Tautologies

Consider the logical expression $(p \lor q) \rightarrow p$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor q$</th>
<th>$(p \lor q) \rightarrow p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

The truth table shows that $(p \lor q) \rightarrow p$ is not a tautology.

For the statement $p \rightarrow q$:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Exercise 4c: Tautologies

\[ \sim (p \& q) \equiv (\sim p \lor \sim q) \]

<table>
<thead>
<tr>
<th>(q)</th>
<th>(p &amp; q)</th>
<th>(\sim (p &amp; q))</th>
<th>(\sim p)</th>
<th>(\sim q)</th>
<th>(\sim p \lor \sim q)</th>
<th>(\sim (p &amp; q) \equiv (\sim p \lor \sim q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

Is a tautology!

<table>
<thead>
<tr>
<th>(p)</th>
<th>(q)</th>
<th>(p \equiv q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Is the conjunction *because* truth-functional?

<table>
<thead>
<tr>
<th></th>
<th>George Bush won the election of 2000</th>
<th>because</th>
<th>Al Gore failed to win key swing states.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td>T</td>
</tr>
</tbody>
</table>

George Bush won the election of 2000 because Al Gore failed to win key swing states.

George Bush won the election of 2000 because The Buccaneers won the Superbowl in 2021.

Not truth functional because the truth of the complex sentence is not a function of the truth of its constituent sentences.