Modality tutorial

http://www-rohan.sdsu.edu/~gawron/optimality

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Overview

1. Introduction
2. Examples
3. Kinds of modality
4. Possible worlds
5. Putting it all together

Outline

Modality

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Goals

- Explain the truth conditions of some complicated sentences in terms of the truth conditions of simpler sentences.

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- Explain the truth conditions of some complicated sentences in terms of the truth conditions of simpler sentences.
- Deal with some of the linguistic variety of modality: Auxiliary verbs, adjectives, adverbs, Conditional sentences
- Understand a bit better what possible worlds are and what they do for us.

Outline

Examples: Necessity

- Necessarily, bachelors are unmarried.
- It is necessarily the case that a bachelor is unmarried.
- It is always the case that a bachelor is unmarried.
- A bachelor must be unmarried.
- A bachelor has to be unmarried.
- # A bachelor is sure to be unmarried.
- A bachelor is unmarried. [Generic reading]

Examples: Possibility

- Possibly, there will be an earthquake tomorrow.
- A triangle may have 3 sides of different lengths.
- It is possible for a man to be older than his own uncle.
- A right triangle may have 3 sides of equal length. [True?]

Logical analysis

Necessarily, bachelors are unmarried	$\Box$unmarried(b)
Squares must be 4-sided.	$\Box$4-sided(s)
Possibly it will rain	$\neg\Diamond$rain()
Right triangles may not have 3 sides of equal length	$\neg\Diamond$3-equal-sides(rt)
Necessarily, right triangles do not have 3 sides of equal length	$\Box\neg$3-equal-sides(rt)

A logical relationship: Duals

$$
\begin{array}{lllll}
1. & \Box \sim p & \iff & \sim \Diamond p \\
2. & \sim \sim \Diamond p & \iff & \sim\sim \Diamond p \\
3. & \sim \Box \sim p & \iff & \sim\sim \Diamond p \\
4. & \sim \Box \sim p & \iff & \Diamond p
\end{array}
$$

$$
\begin{array}{llll}
\Box \sim q & \iff & \sim \Diamond q & \text{Set } q \text{ to } \sim p \\
\Box \sim\sim p & \iff & \sim \Diamond \sim p & \\
\Box p & \iff & \sim \Diamond \sim p &
\end{array}
$$

Outline

Logical

It is logically possible/necessary for p.

- A dog may have 3 legs.
- A triangle must have 3 legs.
- It must either be raining or not raining.
- Mitt Romney might have won the 2012 election (if not for that foolish 47% gaffe).
- Napoleon might have won at Waterloo (if not for the dysentery afflicting his troops).

Epistemic

Given what we know (for all we know), *p*.

- The murderer may have entered through the library.
- The murderer must be right-handed.
- The dinosaurs must have died out suddenly.
- John may go to Cozumel. (possibility in the future)
- John must be in Dubai by now. (necessity in the present)
- Kennedy might not have been shot by Oswald.

Summarizing

- Logical possibilities include those that are contrary to fact.

  She might have fallen down the cliff. Thank god her rescue harness held.

- Epistemic possibilities do NOT include possibilities that are contrary to fact.

  She might have fallen down the cliff. We're still waiting to hear from teh rescire party.

Deontic

Given the rules currently in force, *p*.

- A zombie must be clean and courteous.
- You can smoke only in the designated areas.
- You may have a cookie.
- John has to take Ling 525.

The big picture

	Necessary	Possible
Logical	A bachelor must be unmarried	A triangle may have unequal sides
Epistemic	The murderer must be right-handed	The murderer may have entered here.
Deontic	A zombie must be courteous.	You may smoke here.

Ambiguity

- Kennedy might not have been shot by Oswald.
- Sue may not go to the movies.
- John must be in class.
- A woman might have written the *Odyssey*.

Negation

- The murderer could not have entered through the window.
- The murderer might not have entered through the window.
- John must not be lying.
- John doesn't have to to be lying.

Logical Representations

The murderer could not have entered.	$\neg\Diamond$enter(m)
The murderer might not have entered.	$\Diamond\neg$enter(m)
John doesn't have to be lying.	$\neg\Box$Lie(j)
John must not be lying.	$\Box\neg$Lie(j)

Auxiliaries I

- Modal auxiliaries

(1) a. John may go.[at least two readings]
    b. John must go. [at least two readings]
    c. John can go. [at least two readings]
    d. John should go.
    e. John might go.

- Non "modal" auxiliaries: Distinguish tense, ability

(2) a. John will go.
    b. John can dance. (ability reading)

- Other Modal verbs (not modal auxiliaries, sometimes called "semi-modals")

Auxiliaries II

(3) a. John has to go.
    b. John ought to go.

Outline

Necessity as universal quantification

1. $\Box p$ means: However things might actually turn out to be (or to have been), p is true.
2. We have to talk about p being true or false depending on how things turn out.
3. What does *how things turn out* mean?
4. A **world** is fixed by an assignment of truth values to all atomic statements. That's one way *things turn out*.
5. $\Box p$ means: in all possible worlds p has to turn out to be true.

Necessity: a rigged game

1. But wait. If we can just assign truth values to atomic statements willy nilly how does any p turn otu to be necessarily true?
2. We have postulates (**Meaning postulates**) we must obey:

$$\forall x \text{bachelor}(x) \rightarrow \text{unmarried}(x)$$

3. So we are assigning truth values to atomic statements *in accordance with our knowledge of the entailment properties of English predicates*.
4. Therefore,

$$\Box[\forall x \text{bachelor}(x) \rightarrow \text{unmarried}(x)]$$

The consequences

$$\Box p \iff \forall w[p \text{ is true in } w]$$
$$\sim \Box \sim p \iff \sim \forall w[p \text{ is not true in } w]$$
$$\sim \Box \sim p \iff \sim \forall w \sim [p \text{ is true in } w]$$
$$\Diamond p \iff \sim \forall w \sim [p \text{ is true in } w]$$
$$\Diamond p \iff \exists w[p \text{ is true in } w]$$

Other kinds of modality

Epistemic	Quantifying over all possible worlds consistent what we know, *epistemtically possible worlds* E, w_{E}
Deontic	Quantifying over all possible worlds in which the rules are perfectly obeyed, PO, w_{po}

Outline

Truth definitions I

1. ‘Necessarily, a bachelor is unmarried’ is true iff For all possible worlds $w \in$ W 'a bachelor is unmarried’ is true in w
2. ‘A child could have invented the moustrap’ is true iff There exists some possible world $w \in$ W and 'A child invented the mousetrap’ is true in w
3. 'The lake is sure to freeze tonight’ is true iff for all possible worlds $w \in$ E 'The lake freezes tonight’ is true in w
4. ‘Villagers goats may graze on the green’ is true iff for all possible worlds $w_{po} \in$ PO, 'Villagers goats graze on the green’ is true in w_{po}.

Ambiguity

Negation

Modality assignment I

1. Give truth definitions, giving more than one truth definition with ambiguous examples.
   1. You may not enter the den.
   2. You are not allowed in the den.
   3. You must report to the principal's office.
   4. She might have been arrested.
2. Give logical representations, using predicate logic and $\Box$ and $\Diamond$. For example, 'John must be unmarried.' would be:

$$\Box \sim \text{married(j)}$$

   1. John might not have come early.
   2. The president could not have been in the oval office.