Modality
Jean Mark Gawron
San Diego State University

1 Introduction: Necessity and Possibility

- Modal auxiliaries

  (1) a. John may go.[at least two readings]
       b. John must go. [at least two readings]
       c. John can go. [at least two readings]
       d. John should go.
       e. John might go.

- Non "modal" auxiliaries: Distinguish tense, ability

  (2) a. John will go.
       b. John can dance. (ability reading)

- Other Modal verbs (not modal auxiliaries, sometimes called "semi-modals")

  (3) a. John has to go.
       b. John ought to go.

- Other Modal constructions

  (4) a. Necessarily, John will go.
       b. John will necessarily go.
       c. It is necessary that John go.
       d. It is possible that John will go.
       e. John is required/allowed to go.
       f. A dog always has four legs.[not temporal]

2 Three kinds of Possibility

- Epistemic possibility (consistent with what "we" [the dialogue participants?] know): Generally used in situations of uncertainty (For all we know....)

    – Present epistemic possibility

    (5) a. Careful! There may be some acid left.
    b. The murderer may be a smoker.
    c. The murder may not be a smoker. [both b and c can be true.]
    d. John might be in Cozumel.

    – Past epistemic possibility

    (6) a. They may have used acid.
    b. The murderer may have used a pair of garden shears.
    c. John might have gone to Cozumel.

    – Future epistemic possibility

    (7) John may go to Cozumel.

- Logical Possibility [sometimes also included unde epistemic, especially when *epistemic* is contrasted with *root*]

    – What is asserted may be contrary to what is known: In this case, we assert the *possibility* of something contrary to fact.

    (8) Socrates might have been a woman.

    Yeah, it is historically virtually certain that he wasn't (we have several independent records from contemporrary observers). But the world MIGHT have been different. The teacher of Plato might have been born a woman. Contrast the epistemic case, where there IS uncertainty:

    (9) a. Shakespeare might have been Francis Bacon.
    b. Shakespeare might have been Edward de Vere, Earl of Oxford. [Uncertainty: both (a) and (b) can be true at the same time.]
    c. Homer might have been a woman.

– What is asserted may be general. In this case, we assert the logical *possibility* of something which may or may not be the case.

(10) A man may own two cars.

– *May* vs *might*: For many speakers (our text asserts this may be changing), *may have* can only be used with epistemic modality.

(11) a. Shakespeare may have been Edward de Vere, Earl of Oxford.
 b. Homer may have been a woman.
 c. ? Socrates may have been a woman. [? reading: Things might have turned out so that Socrates had been born a woman.]

- Deontic Possibility (Permission)

(12) a. You may have 3 cookies.
 b. You may not smoke here.
 c. You are not permitted/allowed to smoke here.
 d. Do not smoke! [Related to the above, doesnt really belong on this list. A different speech act!]

3 Three kinds of Necessity

- Epistemic and Logical Necessity, Deontic Obligation

 – Epistemic: True in any circumstances consistent with what we know.

(13) Britney Spears must drink (a lot of) Pepsi. (uttered by a naive commercial watcher)

 – Logical: True in any circumstances whatsoever

(14) A square must have 4 sides.

 – Deontic: True in any circumstances in which ethical canons are satisfied.

(15) Britney Spears must drink (a lot of) Pepsi. (uttered by a hard-line Pepsi attorney)

- Logical vs Epistemic Necessity

  Logical

  (16) a. 2 + 2 must equal 4.
        b. Parallel lines may not intersect.

  Epistemic

  (17) a. The dinosaurs must have died out suddenly.
        b. The director must have been mistaken.

  This distinction is often a matter of discussion for philosophers.

  (18) a. Force must be directly proportional to acceleration. [Newton's
           2nd Law of Motion. Is this a law we've deduced or a definition
           of force?]
        b. A bachelor is unmarried.
        c. A dog is a mammal.
        d. A beech is a tree.
        e. A beech has nuts.

4   Basic Dimensions

- The dimensions

  (19)

	Logical	Epistemic	Deontic
Necessity/ Obligation	must has to always necessarily	must has to	must has to ought to
Possibility/ Permission	might can possibly	may/might can possibly	may can possibly

4

5   Scope

- Examples from Walter Cook, S.J.

Sentence	Reading	
John can't be lying	$=$	It is not possible that John is lying
	$\neq$	It possible that John is not lying
John cannot be lying	$\neq$	It is not possible that John is lying
	$=$	It possible that John is not lying
John must not be lying	$\neq$	It is not necessary that John is lying
	$=$	It necessary that John is not lying
John may not be lying	$\neq$	It is not possible that John is lying
	$=$	It possible that John is not lying
John doesn't have to be lying	$=$	It is not necessary that John is lying
John does not have to be lying	$\neq$	It necessary that John is not lying

- Logical Representations

John can't be lying.	$\neg\Diamond\text{Lie(j)}$
John may not be lying.	$\Diamond\neg\text{Lie(j)}$
John doesn't have to be lying.	$\neg\Box\text{Lie(j)}$
John must not be lying.	$\Box\neg\text{Lie(j)}$

6   Connection to Conditionals

- Each of the types of modality occurs in conditionals

  A deontic example

  (20)   If school tradition requires that the champions and their partners
         open the ball, and if Harry is champion, then Harry must dance.

(21)

Logical	Necessity	(a)	If the angles of a triangle are all 60°, then it must be equilateral.
		(b)	If John had been a little quicker, then he would still be alive.
	Possibility	(c)	If two lines are coplanar, then they may be parallel.
		(d)	If John had been a little quicker, then he could still be alive.
Epistemic	Necessity	(e)	If the seat is warm, then the murderer must still be in the den.
		(f)	If the seat is warm, then the murderer will still be in the den.
	Possibility	(g)	If the seat is warm, then the murderer may still be in the den.
		(h)	If the seat is warm, then the murderer will still be in the den.
Deontic	Obligation	(i)	If John made a promise, then he must come to the party.
		(j)	If John made a promise, then he should come to the party.
	Permission	(k)	If the admiral grants their leave requests, then the cadets may come to the ball.
		(l)	If the admiral granted their leave requests, then the cadets could come to the ball.

- Conclusion The *will,would,should* of conditionals are modal auxiliaries parallel to *must*; and conditional *could* is parallel to *can*.

(22)

	Logical	Epistemic	Deontic
Necessity/ Obligation	must **will, would** has to	must **will** has to	must **should** has to
Possibility/ Permission	might can **could**	may/might can **could**	may can **could** ought to

7 Possible worlds

- A logically necessary proposition is true in any circumstances whatsoever

(23) a. | Logically necessary modal claims are true iff they are true in all possible worlds. [true in all $w \in W$, the set of all possible worlds.]

  b. | Logically possible modal claims are true iff they are true in some possible world. [true in some $w \in W$, the set of all possible worlds.]

(24) a. A triangle must have 3 sides.

$$\forall w \in W[\text{3-sided(triangles) in } w]$$
$$\Box[\text{3-sided(triangles)}]$$

  b. A man may own two cars.

$$\exists w \in W \exists x[\text{man}(x) \text{ in } w \land \text{2-car-owner}(x) \text{ in } w]$$
$$\Diamond \exists x[\text{man}(x) \land \text{2-car-owner}(x)]$$

- An epistemically necessary proposition is true in those circumstances consistent with what we know

(25) a. | Epistemically necessary modal claims are true iff they are true in all worlds consistent with what we know. [true in all $w \in E$, the set of all epistemically possible worlds.]

  b. | Epistemically possible modal claims are true iff they are true in some epistemically possible world. [true in some $w \in W$, the set of all possible worlds.]

(26) a. John must be lying.

$$\forall w \in E[\text{Lie}(j) \text{ in } w]$$
$$\Box[\text{Lie}(j)]$$

  b. John may be lying.

$$\exists w \in E[\text{Lie}(j) \text{ in } w]$$
$$\Diamond[\text{Lie}(j)]$$

- A deontically necessary proposition is true in those circumstances in which all ethical canons are obeyed.

(27) a. | Deontically necessary modal claims are true iff they are true in all worlds obedient to all ethical standards. [true in all $w \in$ D, the set of all deontically perfect worlds.]

   b. | Deontically possible modal claims are true iff they are true in some world obedient to all ethical standards. [true in some $w \in$ D, the set of all deontically perfect worlds.]

- Logical Representation of scope carries over

John can't be lying.	$\neg \exists w \in \text{E}[\text{Lie(j) in } w]$
John may not be lying.	$\exists w \in \text{E}\neg[\text{Lie(j) in } w]$
John doesn't have to be lying.	$\neg \forall w \in \text{E}[\text{Lie(j) in } w]$
John must not be lying.	$\forall w \in \text{E}\neg[\text{Lie(j) in } w]$

8 A nice feature of the possible worlds account

- Necessary modal claims are universal quantifications over worlds; Possible modal claims are existential quantifications over worlds.

- Some truths of logic

$$(a) \quad \forall x \text{grin}(x) \iff \neg \exists x \neg \text{grin}(x)$$
$$(b) \quad \exists x \text{grin}(x) \iff \neg \forall x \neg \text{grin}(x)$$

- $\forall$ and $\exists$ are called logical **duals**

- This means if our possible worlds treatment is correct, **necessary** ($\Box$) and **possible** ($\Diamond$) ought to be duals too.

(28) a. John must grin $\iff$ It is not the case that it is possible that John not grin.

$$\forall w \in \text{W}[\text{grin}(j) \text{ in } w] \iff \neg \exists w \in \text{W}\neg[\text{grin}(j) \text{ in } w]$$

b. John may grin $\Longleftrightarrow$ It is not the case that it is necessary that John not grin.

$$\exists w \in W[\text{grin}(j) \text{ in } w] \Longleftrightarrow \neg\forall w \in W\neg[\text{grin}(j) \text{ in } w]$$

- *be obligated to* and *be allowed to* are duals

(29) a. John has to go $\Longleftrightarrow$ It is not the case that John is permitted not to go.
  b. John is allowed to go $\Longleftrightarrow$ It is not the case that John is obligated not to go.

9 Rethinking the 3 kinds of modality: Context and Modals

- Context may provide sentences with modal auxiliaries a covert "antecedent" (Kratzer 1977)

  a "deontic" modal example

(30) a. According to the school's tradition, the champions and their partners open the ball.
  b. Harry is champion.
  c. **Harry must dance.**
  d. [In context =]: If school tradition requires that the champions and their partners open the ball, and if Harry is champion, then Harry must dance.
  e. Version I: In all worlds w in which Harry is champion in w and $w \in D$ (the deontically perfect worlds), Harry dances in w.

- The problem with perfect worlds

  – But we know perfectly well that sometimes ethical principles conflict.

  – Suppose in some world w school tradition is ignored in order that the money spent on a massively expensive Christmas ball may instead be spent on Christmas gifts for the children at the orphanage. [Awwwww...]

- Arguably w, a world in which Harry doesnt dance because there is no ball, is an ethically more perfect world than w', a world in which the ball is put on and Harry dances.

- But if w is an ethically ideal world **Harry must dance** comes out false because there is an ethically ideal world in which Harry doesn't dance.

• The solution

(31) We *order* worlds according to how well they satisfy the set of ethical principles **relevant in context**

revised "deontic" modal example

(32) a. In the context of (30), it appears as if the ethical principle most relevant for ordering worlds is adhering best to school tradition.

b. Version II: For all worlds w in which Harry is champion and which correspond most to tradition, Harry dances in w.

• Moral: Two kinds of constraints on worlds are relevant in evaluating modals

- Modal base: A set of worlds satisfying the "antecedent" conditions (the worlds in which Harry is champion in our example)

- An ordering on those worlds, which in the previous example was deontic in nature. The ordering reflects which propositions are most relevant (in context) as ethical principles (the principle of tradition in our example).

10 Other Dimensions

• Buletic (worlds consistent with desires)

(33) If you want to go to Harlem, you must take the A train. [new kind of modality]

Wrong truth-conditions: In all worlds in which you want to go to Harlem, you take the A train.

Right truth-conditions: In all worlds in which you want to go to Harlem, *and your desires are fulfilled*, you take the A train.

- Evidentiality: Something more to epistemic *must*

(34) a. The toast must be burning.
      b. The toast is burning.
      c. (b) more natural than (a) in the following context: You have just put the toast in the toaster and you are standing in the kitchen watching black smoke pour out of the toaster.
      d. (a) more natural than (b) in the following context: You have just put the toast in the toaster and you are standing in the living room and you smell something burning.
      e. *must* ϕ is used when the evidence for ϕ is indirect, but ϕ can be inferred from that evidence.