Logical Translations

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1 Introduction

We introduce the connectives of **statement logic**.

We introduce predicates. And a very simple semantics for them.

We introduce important **semantic relations** among sentences: entailment, contraries, contradictories. We introduce related properties of single sentences: logical truth (tautology), contradiction.

2 Truth-Functional Connectives

2.1 And

Consider our extension rule for 'and'

$$[\![\text{A and B}]\!] = \text{True if and only if } [\![\text{A}]\!] = \text{True and } [\![\text{B}]\!] = \text{True.}$$

	A	B	A and B
(a)	T	T	T
(b)	T	F	F
(c)	F	T	F
(d)	F	F	F

This says the same as the rule for & in the text book:

	p	q	p & q
(a)	T	T	T
(b)	T	F	F
(c)	F	T	F
(d)	F	F	F

Idea: Use & as the **logical translation** for *and*:

(1) a. Abraham Lincoln was elected in 1860 and he was re-elected in 1864.

b. John picked up the apple and he ate it.
c. ? John ate the apple and he picked it up. [temporal order, pragmatics?]
d. You take one more step and I'll shoot. [= If you take one more step, I'll shoot]

2.2 Or

	p	q	$p \vee q$
(a)	T	T	T
(b)	T	F	T
(c)	F	T	T
(d)	F	F	F

(2) a. Either he rented either a mid-size car or he rented an economy car. [$p \vee q$ translation claims: If in fact he rented both, this is still true]
b. Either there's no bathroom in this house or it's on the second floor. [In fact both statement can't be simultaneously true.]
c. You can have either the white one or the red one. [intended meaning: but not both]

Exclusive or

	p	q	$p \overline{\vee} q$
(a)	T	T	F
(b)	T	F	T
(c)	F	T	T
(d)	F	F	F

Predictions of exclusive-or analysis:

(3) Maria is very smart, or she is very hard-working.

Suppose Maria is both.

2.3 Material implication

Material implication is the name we'll use for $\rightarrow$.

	p	q	$p \rightarrow q$
(a)	T	T	T
(b)	T	F	F
(c)	F	T	T
(d)	F	F	T

(4) a. If Alice wins a fellowship, she will finish her thesis.
b. Antecedent: Alice (will win)/wins a fellowship.
c. Consequent: Alice will finish her thesis.

> Claim A
> In those circumstances where the first sentence (**the antecedent**) is true, the second sentence (**the consequent**) is true.

So the first two lines of the truth table make perfect sense. Claim A is safe when both sentences are true, and it is clearly false when the antecedent is true and the consequent is false.

But what about when the first sentence is false? Here's what we're going to argue: if Alice doesn't win a fellowship, claim A is safe whether she finishes her thesis or not. Claim A only requires that IF she wins the fellowship, thesis-finishing follows. So if she didn't, the claim is still compatible with the facts ("true"), according to the truth table.

One case to consider: Suppose she doesn't win a fellowship. Does asserting (4a) commit us to the claim that she doesn't finish her thesis? [No. Suppose a millionaire donor gives her enough money to stop working and finish that thesis. (4a) doesn't rule that out. It just says that a fellowship would give her enough money to finish. It doesn't pteclude otehr routes.]

Question: How well does this truth table accord with our intutuions about conditional sentences in English in general (*if ... then ...)* ? Answer: Not very.

(5)

	Ant.	Cons.	Conditional	Truth Value
(a)	T	T	If 1960 was divisible by 5, then 1960 was a leap year.	T
(b)	F	T	If Al Gore won the election of 2000, George Bush won the election of 2004.	T
(c)	T	F	If George Bush won the election of 2004, Al Gore won the election of 2000.	F

(a) just seems false. (b) is weird not clear what kind of communicative act is being performed. (c) can be true as an instance of the "If X, I'll eat my hat" construction.

3 Relations among Sentences

3.1 Contraries

Two sentences are contraries if they can't both be true at the same time.

(6) a. John is tall.
b. John is short.

(7) a. John is happy.
b. John is sad.

(8) a. This is a triangle.
b. This is a square.

(9) a. Fido is a dog.
b. Fido is a country in Eastern Europe.

3.2 Contradictories

Two sentences are contradictories if they are contraries and can't both be false.

(10) a. John is a fool.

b. John is not a fool.

(11) a. Some dog barked.
b. No dog barked.

Not contradictories, only contraries.

(12) a. John is tall.
b. John is short.

What about?

(13) a. John is very tall.
b. John is not very tall.

3.3 Entailment

1. A sentence A entails a sentence B if whenever A is true, B has to be true.

2. A sequence of sentences A_1, A_2, A_n entail a sentence B if whenever A_1, A_2, A_n are true, B has to be true.

Thanks to Gennaro Chierchia and Sally McConnell-Ginet for the following examples.

The first two sentences entail the second.

(14) a. This is yellow.
b. This a fountain pen.
c. This is a yellow fountain pen.

How about these?

(15) a. This is big.
b. This a sperm whale.
c. This is a big sperm whale.

(16) a. Lee kissed Kim passionately.
b. Lee kissed Kim.

c. Kim was kissed by Lee.
Kim was kissed.
Lee touched Kim with her lips.

Also consider this: Does Lee mouthing a kiss at Kim from 15 feet away count as kissing him?

(17) a. Jane ate oatmeal for breakfast today.
b. Jane ate breakfast today.

(18) a. Juan is not aware that Mindy is pregnant.
b. Mindy is pregnant.

(19) a. Juan does not believe that Mindy is pregnant.
b. Mindy is pregnant.

(20) a. Juan believes that Mindy is pregnant.
b. Mindy is pregnant.

(21) a. Every second-year student who knows Latin will get credit for it.
b. If Juan is a second-year student and knows Latin, he will get credit for it.

(22) a. If Alice wins a fellowship, she can finish her thesis.
b. If Alice doesn't win a fellowship, she can't finish her thesis.

(23) a. If Alice finished her thesis, then she won a fellowship.
b. If Alice didn't win a fellowship, then she didn't finish her thesis.

(24) a. Maria and Alberto are married.
b. Maria and Alberto are married to each other.

(25) a. Only Amy knows the answer.
b. Amy knows the answer.

(26) a. Amy knows the answer.
b. Only Amy knows the answer.

(27) a. Mary is a Italian violinist.
b. Some Italian is a violinist.

c. Some violinist is Italian.

(28) a. Some student will not go to the party.
b. Not every student will go to the party.

(29) a. Allegedly, John is a good player.
b. John is a good player.

(30) a. Oscar and Jenny are rich.
b. Oscar is rich.
c. Jenny is rich.

(31) a. Oscar and Jenny are middle-aged
b. Oscar is middle-aged.
c. Jenny is middle-aged.

3.4 Logical equivalence: The logical importance of and

(32) a. Mary is an Italian violist.
b. Mary is a violinist.
c. Mary is Italian.
d. Mary is a violinist and Mary is Italian.

(33) a. This is a yellow fountain pen.
b. This is yellow and this is a fountain pen.

(34) a. John and Mary ate.
b. John ate.
c. Mary ate.
d. John ate and Mary ate.
e. John or Mary ate.
f. John ate or Mary ate.

(35) a. John and Mary ate
b. p = John ate.
c. q = Mary ate.
d. $[\![$John and Mary ate. $]\!] = p \,\&\, q$.
e. eat(John) & eat(Mary)

(e) captures how the individual words contribute to the meaning.

4 Ambiguity

(36) a. You should have seen that bull we got from the pope.
b. Competent women and men hold all the good jobs in the firm.
c. Mary claims that John saw her duck.
d. John and Mary are married.

Ambiguous sentences have more than one meaning. That means if we translating into an unambiguous logical language, they should get more than one translation. This is one of the ways in which the logical analysis is a help, it gives us a tool to spell out what the distinct readings are and maybe why they exist.

Consider (36d), a case we are going to want to call ambiguous, but which not seem so obviously a case of ambiguity at first.

Pattern 1 (very general)

(37) a. A is Adj.
b. B is Adj.
c. A and B are Adj.
d. happy, tall, incompetent, married, annoying,...

Pattern 2 (symmetric predicates)

(38) a. A is Adj Prep B.
b. A and B are Adj.
c. similar to, married to, adjacent to, different from

Unambiguous

(39) a. John and Mary are annoying.
b. John and Mary are similar.

Question: Why is (b) unambiguous? How does *similar to* differ from *married to*?

Revised view?

(40) a. Maria and Alberto are married.
b. Maria and Alberto are married to each other.