Quantifier tutorial
http://www-rohan.sdsu.edu/~gawron/semantics

Jean Mark Gawron

San Diego State University, Department of Linguistics

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Overview

1. Introduction

2. Answers

3. Background ideas
   - General principles
1 Introduction

2 Answers

3 Background ideas
   • General principles
Logical Form

Goal:

- A few simple rules to help the beginner get the hang of translating into logic
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- Problems
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Problems

- There are a LOT of things to cover
- The rules can’t be complete.
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- Problems
  - There are a LOT of things to cover
  - The rules can’t be complete.
  - Ambiguity of English
Logical translations (I)

Alice passed every message to Ralph.

\[
[\text{every } x \ \text{message}(x)] \text{pass}(a, x, r)
\]

Anne has read most books on psychoanalysis

\[
[\text{most } x \ \text{book}(x) \land \text{on}(x, p)] \text{read}(a, x)
\]

Few who knew him supported Baxter.

\[
[\text{few } x : \ \text{person}(x) \land \text{know}(x, b)] \text{support}(x, b)
\]

Morris shared every fingerprint he dusted

\[
[\text{every } x : \ \text{fingerprint}(x) \land \text{dust}(m, x)] \text{share}(m, x)
\]
Marcia peeled and quartered three apples.

\[ \text{three } x : \text{apple}(x) \] \text{peel}(m, x) \land \text{quarter}(m, x) \]

Marcia and Clive ate four apples each. (handling each)

\[ \text{four } x : \text{apple}(x) \] \text{eat}(m, x) \land \text{eat}(c, x) \]

Ramon signs every sculpture he makes.

\[ \text{every } x : \text{sculpture}(x) \land \text{make}(r, x) \] \text{sign}(r, x) \]

Jones restored and sold several valuable paintings.

\[ \text{several } x : \text{painting}(x) \land \text{valuable}(x) \] \text{restore}(j, x) \land \text{sell}(j, x) \]
Most bulbs will not grow if they are dry.

$$[\text{Most } x : \text{bulb}(x) \land \text{dry}(x) ] \sim \text{grow}(x)$$

The witch(def. w) picked a leaf from every tree in the forest (def. f).

$$[\text{every } x : \text{tree}(x) \land \text{in}(x, f) ] [A \ x : \text{leaf}(x)] \text{pick}(w, x, y)$$

A few people from each town lost everything they owned (3 Quantifiers)

$$[\text{every } x : \text{town}(x)]$$
$$[\text{a-few } y : \text{people}(y) \land \text{from}(y, x) ]$$
$$[\text{every } z : \text{thing}(z) \land \text{own}(y, z) ] \text{lose}(y, z)$$
Everyone in this room (def. $r$) speaks two languages. (*everyone is bilingual*)

\[
\forall x : \text{person}(x) \land \text{in}(x, r) \ [\exists y : \text{language}(l) \ [\text{speak}(x, l)]
\]

Everyone in this room (def. $r$) speaks two languages. (*There are two languages that everyone in this room speaks*)

\[
\exists y : \text{language}(l) \ [\forall x : \text{person}(x) \land \text{in}(x, r) \ [\text{speak}(x, l)]
\]
Most boys like most girls

\[
[\text{Most } x : \text{boy}(x) ][\text{Most } y : \text{girl}(y) ] \text{like}(x, y)
\]

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Most boys like most girls

\[ \text{Most } x : \, \text{boy}(x) \ [\text{Most } y : \, \text{girl}(y) \ [\text{like}(x, y) \] \]

\[ \text{Most } y : \, \text{girl}(y) \ [\text{Most } x : \, \text{boy}(x) \ [\text{like}(x, y) \] \]
Are both readings true?

(a) \[\text{Most } x : \text{boy}(x) \] \[\text{Most } y : \text{girl}(y) \] \text{like}(x, y)
(b) \[\text{Most } y : \text{girl}(y) \] \[\text{Most } x : \text{boy}(x) \] \text{like}(x, y)
Just two of the ten arrows are broken. (A is the set of arrows (in context), B the set of broken things)

\[ |A \cap B| = 2 \text{ and } |A| = 10 \]

Between 5 and 10 airlines are bankrupt. (A is the set of airlines, B the set of bankrupt entities)

\[ 5 \leq |A \cap B| \leq 10 \]

Both avenues are broad. (A is the set of avenues, B the set of broad things, which includes the shoulders of Shaquille O’Neal)

\[ |A \cap B| = 2 \text{ and } |A| = 2 \]

or

\[ A \subseteq B \text{ and } |A| = 2 \]
Fewer than 5 aubergines are baked. (A is the set of aubergines, B the set of baked things)

\[ |A \cap B| < 5 \]
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Outline

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The **logical form** of an English sentence is a **decomposition** of the sentence into **predicates** and **connectives**. The predicates capture the concepts being expressed. The connectives capture how the concepts are related.

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   1. Same arity (same number of arguments)
   2. Arguments are in a consistent order
Operators and Connectives: Quantifiers, negation, and sentential (FOL)

Universals ($\forall$), Existentials ($\exists$), and negation $\sim$ correspond to appropriate English words, and each quantifier goes with its appropriate sentential connective:

<table>
<thead>
<tr>
<th>every, all, any</th>
<th>$\forall$</th>
<th>$\forall x$ dog($x$) $\rightarrow$ bark($x$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>some, a, a certain</td>
<td>$\exists$</td>
<td>$\exists$ &amp; $\exists x$ dog($x$) &amp; bark($x$)</td>
</tr>
<tr>
<td>not, n’t</td>
<td>$\sim$</td>
<td>$\sim \exists$ &amp; $\sim \exists x$ dog($x$) &amp; bark($x$)</td>
</tr>
<tr>
<td>no</td>
<td>$\sim \exists$</td>
<td></td>
</tr>
<tr>
<td>every dog, every button hook</td>
<td>[every ( x ) : dog ], [every ( x ) : buttonhook ]</td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td>--------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>most dogs, most button hooks</td>
<td>[most ( x ) : dog ], [most ( x ) : buttonhook ]</td>
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<td>[no ( x ) : dog ], [no ( x ) : buttonhook ]</td>
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Some students who heard both concerts were interviewed by Holmes.
[Some \( x \) student\((y) \) &
  
  \([\text{Both} \ y \ \text{concert}(y) \ \& \ \text{hear}(x, \ y)] \) ]

  \text{interview}(H, \ x)

Some students who heard both concerts were interviewed by Holmes.
[\text{Both} \ y \ \text{concert}(y) ] [\text{Some} \ \times \ \text{student}(y) \ \&

  \& \ \text{hear}(x, \ y)]

  \text{interview}(H, \ x)