Property Definitions: Russell

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**Sets defined by properties: I**

Set of elephants

\[ \text{ELEPHANT} = \{ x \mid \text{elephant}(x) \} \]

\text{ELEPHANT} contains no sets. It particular, it does not contain itself!

\[ \text{ELEPHANT} \notin \text{ELEPHANT} \]
Sets defined by properties: II

Set of abstract concepts

\[ \text{AC} = \{ x \mid \text{abstract-concept}(x) \} \]

AC contains itself!

\[ \text{AC} \in \text{AC} \]
The Russell property

\[ U = \{ x \mid x \not\in x \} \]
The paradox

\[ U = \{ x \mid x \notin x \} \]

1. Either \( U \in U \) or \( U \notin U \). (Definition of set)
2. If \( U \in U \) then \( U \notin U \). (Definition of \( U \)). Contradiction!
3. If \( U \notin U \) then \( U \in U \). (Definition of \( U \)). Contradiction!
Grelling’s Paradox

\[ \text{AUTOLOGICAL} = \{ x \mid x \text{ is an adjective that describes itself} \} \]

\[ \text{HETEROLOGICAL} = \{ x \mid x \text{ is an adjective that does not describe itself} \} \]

1. The adjective *short* is autological.
2. The adjective *long* is heterological.
3. What about the adjective *heterological*?
4. What about the adjective *autological*?