Rules, Functions, and Recursive Definitions

1 Rules as functions

Consider the English past tense, phonetically. A citation form is the form we find in the dictionary. The citation form of walk is /wɔk/. The past tense form is /wɔkt/ (spelled walked). So you add /t/:

If \( \alpha \) is a verb citation form, then \( \alpha + /t/ \) is the past tense of the verb.

There are three problems with this. It doesn’t work for verbs that end with voiced sounds; the past tense of hug is not /hʌgt/; it’s /hʌgd/. It doesn’t work for verbs that end with /d/ or /t/. The past tense of raid is /redəd/ and the past tense of sight is /saɪtəd/. Finally, it doesn’t work for irregular verbs. The past tense of sing is not /sɪŋt/. It’s /sæŋ/ (spelled sang).

Let’s leave the irregulars out of it and fix the regulars:

1. If \( \alpha \) is a regular verb citation form, and \( \alpha \) ends in /t/ or /d/, then \( \alpha + /d/ \) is the past tense of the verb; otherwise,

2. if \( \alpha \) is a verb citation form, and \( \alpha \) ends in a voiceless sound, then \( \alpha + /t/ \) is the past tense of the verb;

3. otherwise, if \( \alpha \) is a verb citation form, then \( \alpha + /d/ \) is the past tense of the verb.

This defines a function. Let’s call the function \( \text{Past} \) and let’s call the set of regular verb citation forms \( \text{Verb}_{reg} \). Here’s how the function definition looks in our textbook’s notation:

\[
\text{Past} = \{ (\alpha, \alpha + \text{suf}) \mid \alpha \in \text{Verb}_{reg} \text{ and } \text{suf} = /d/ \text{ if END}(\alpha) \in \{/t/, /d/\}; \text{ and } \\
\text{suf} = /t/ \text{ if END}(\alpha) \in \text{Voiceless}; \text{ and } \\
\text{suf} = /d/ \text{ otherwise} \}
\]

I’m assuming that \( \text{END} \) is itself a function that for each verb stem, returns the last sound in it. So, for example:

\[
\text{END}(/wɔk/) = /k/
\]
Given this definition of \textbf{Past}, it’s now legitimate to write:

\begin{align*}
\text{Past}/\text{red}/ &= \text{red}d \\
\text{Past}/\text{wOk}/ &= \text{wOk}t \\
\text{Past}/\text{h2g}/ &= \text{h2g}d
\end{align*}

\section{Reviewing Recursive Definitions}

Defining the Natural numbers:

\begin{enumerate}
\item \(0 \in \mathbb{N}\)
\item \(\text{If } x \in \mathbb{N} \text{ then } \text{successor}(x) \in \mathbb{N}\)
\item \(\mathbb{N}\) is the smallest set that satisfies clause (i) and (ii)
\end{enumerate}

Is 3 a natural number....?

1. 0 is a natural number. (Axiom i).
2. \text{successor}(0)=1. (Def of successor function)
3. 1 is a natural number (Axiom ii on steps 1 and 2)
4. \text{successor}(1)=2. (Def of successor function)
5. 2 is a natural number (Axiom ii on steps 3 and 4)
6. \text{successor}(2)=3. (Def of successor function)
7. 3 is a natural number (Axiom ii on steps 5 and 6) Q. E. D.

This is called a recursive definition. In order to define what’s in the set \(\mathbb{N}\) I make reference to what’s in the set \(\mathbb{N}\) (clause ii).

The official Definition of \(\mathbb{N}\) (Peano’s definition):

\begin{enumerate}
\item \(0 \in \mathbb{N}\)
\item \(\text{If } x \in \mathbb{N} \text{ then } \text{successor}(x) \in \mathbb{N}\)
\item \(\mathbb{N}\) is the smallest set that satisfies clause (i) and (ii)
\end{enumerate}

Sufficient and necessary conditions required. Clause (i) and (ii) alone aren’t enough to keep Bill Clinton out of the set of natural numbers.
Recursive definitions of grammars

Definition of a small language recursively by defining the set of sentences of the language, which we call \( S \).

I’ll use ‘\( xy \)’ to mean ‘\( x \)’ followed by (or concatenated with) ‘\( y \)’:

1. First we define a set \( N \) as follows:
   
2. Let \( N = \{ \text{book, magazine, boy, girl} \} \)
3. Let \( \text{Adj} = \{ \text{big, fat} \} \)
4. Let \( V = \{ \text{liked, loved} \} \)
5. Let \( \text{Art} = \{ \text{the} \} \).
6. Next we define a set \( \text{Nom} \):
   
   (a) If \( x \in N \) then \( x \in \text{Nom} \).
   
   (b) If \( x \in \text{Adj} \) and \( y \in \text{Nom} \) then \( xy \in \text{Nom} \).
   
   (c) Nothing else is in \( \text{Nom} \).
7. Next we define a set \( \text{NP} \), using \( \text{Nom} \):
   
   (a) If \( x \in \text{Art} \) and \( y \in \text{Nom} \) then \( xy \in \text{NP} \).
   
   (b) Nothing else is in \( \text{NP} \).
8. Next we define a set \( \text{VP} \):
   
   (a) If \( x \in V \) and \( y \in \text{NP} \) then \( xy \in \text{VP} \).
   
   (b) Nothing else is in \( \text{VP} \).
9. Finally we define the set \( S \), the set of sentences of the language.
   
   (a) If \( x \in \text{NP} \) and \( y \in \text{VP} \) then \( xy \in S \).
   
   (b) If \( x \in \text{NP} \) and \( y \in S \) then ‘\( x \) believed \( y \)’ \( \in S \).
   
   (c) Nothing else is in \( S \).

Questions

1. How would a linguist define this language using phrase structure rules?
2. Is $S$ infinite?

3. Which of the following are in $\text{Nom}$?

   - boy
   - big boy
   - big fat boy
   - fat big boy
   - the big fat boy

4. Is the following in $S$?

   The big boy believed the girl believed the boy liked the big fat book.